hamburger sternwarte

Evolution of Primordial Magnetic Fields

Robi Banerjee

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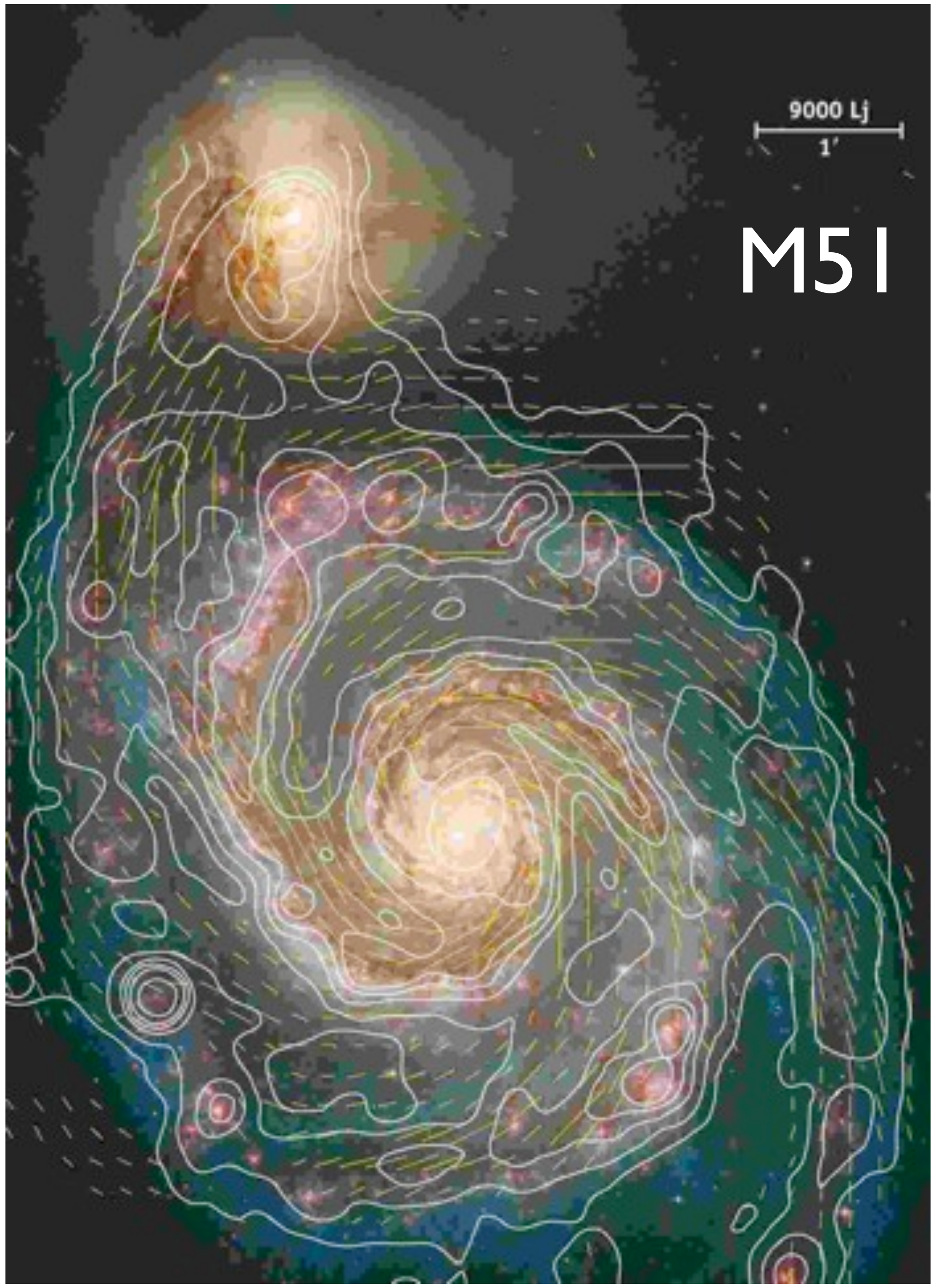
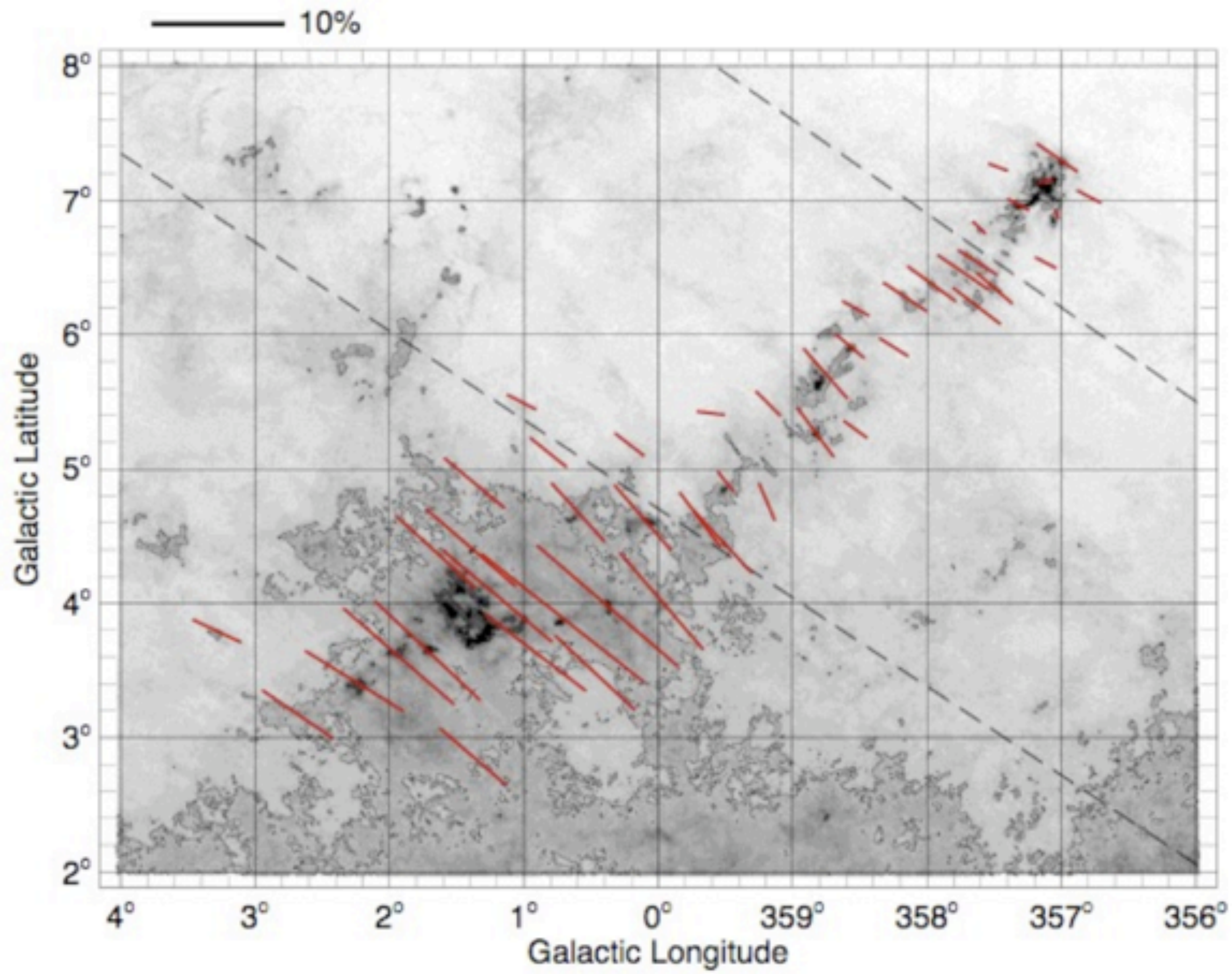
Observed Magnetic Fields



galactic B-fields (e.g. *R.Beck 2001*)
large scale component: ~ 4μG
total field strength: ~ 6μG

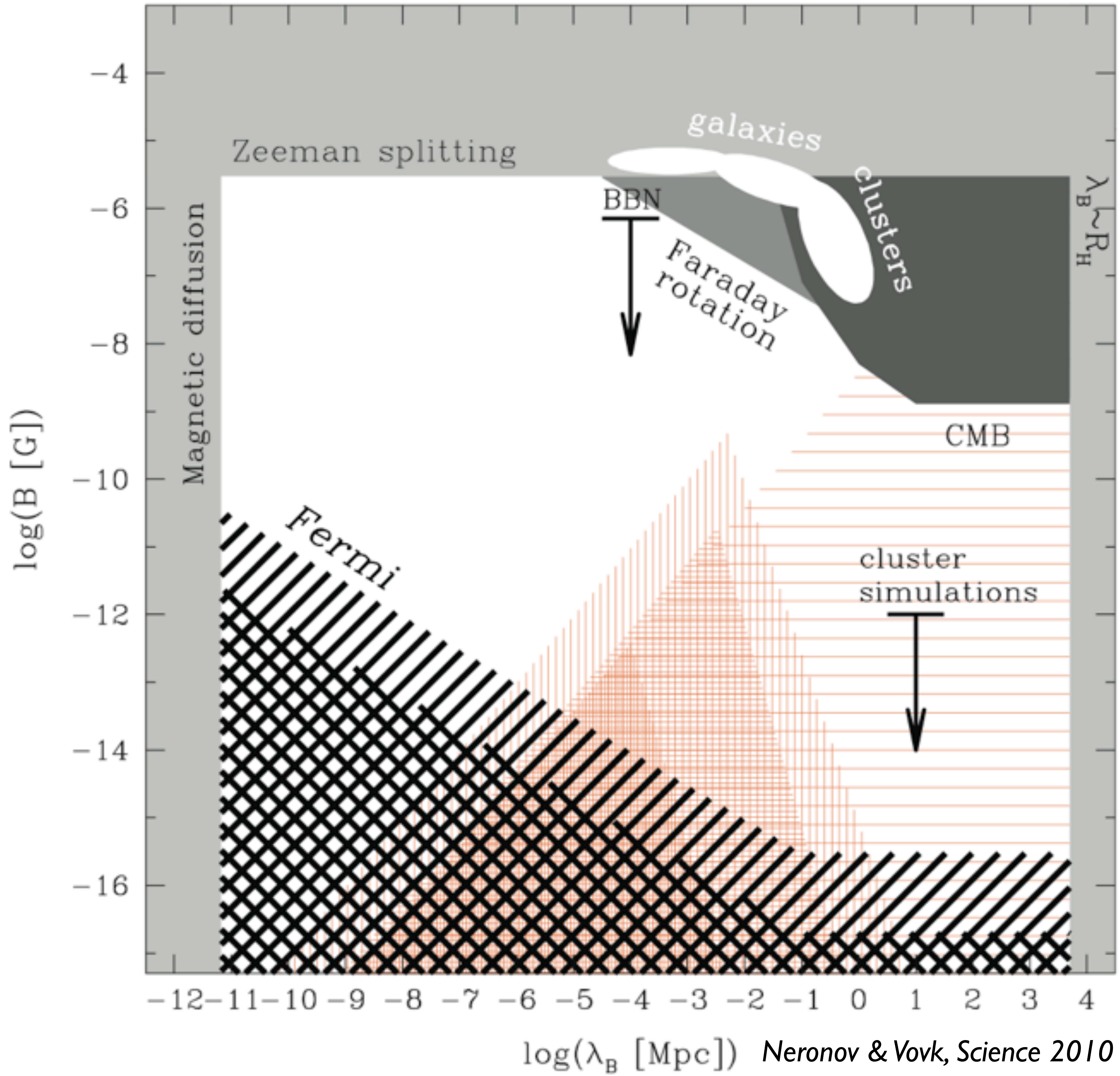
Magnetic fields are observed on all scales



magnetic polarization measurements in the Pipe nebula
F.O.Alves, Franco, Girart 2008

Cosmic Magnetic Fields

- Observations:
  - Galactic fields ~ few μG
    (e.g. Beck 1999)
  - Cluster fields ~ μG
    (e.g. Bonafede et al. 2010)
- Upper limits:
  - BBN ~ 10^{-7} G
    (Grasso & Rubinstein 2001)
  - CMBR ~ 10^{-9} G
  - Reionization ~ 10^{-9} G
    (Schleicher et al. 2008)
- **Lower** limits ~ 10^{-15} G
  (FERMI obs. e.g. Neronov & Vovk 2010; Taveccio et al. 2010)



Neronov & Vovk, Science 2010

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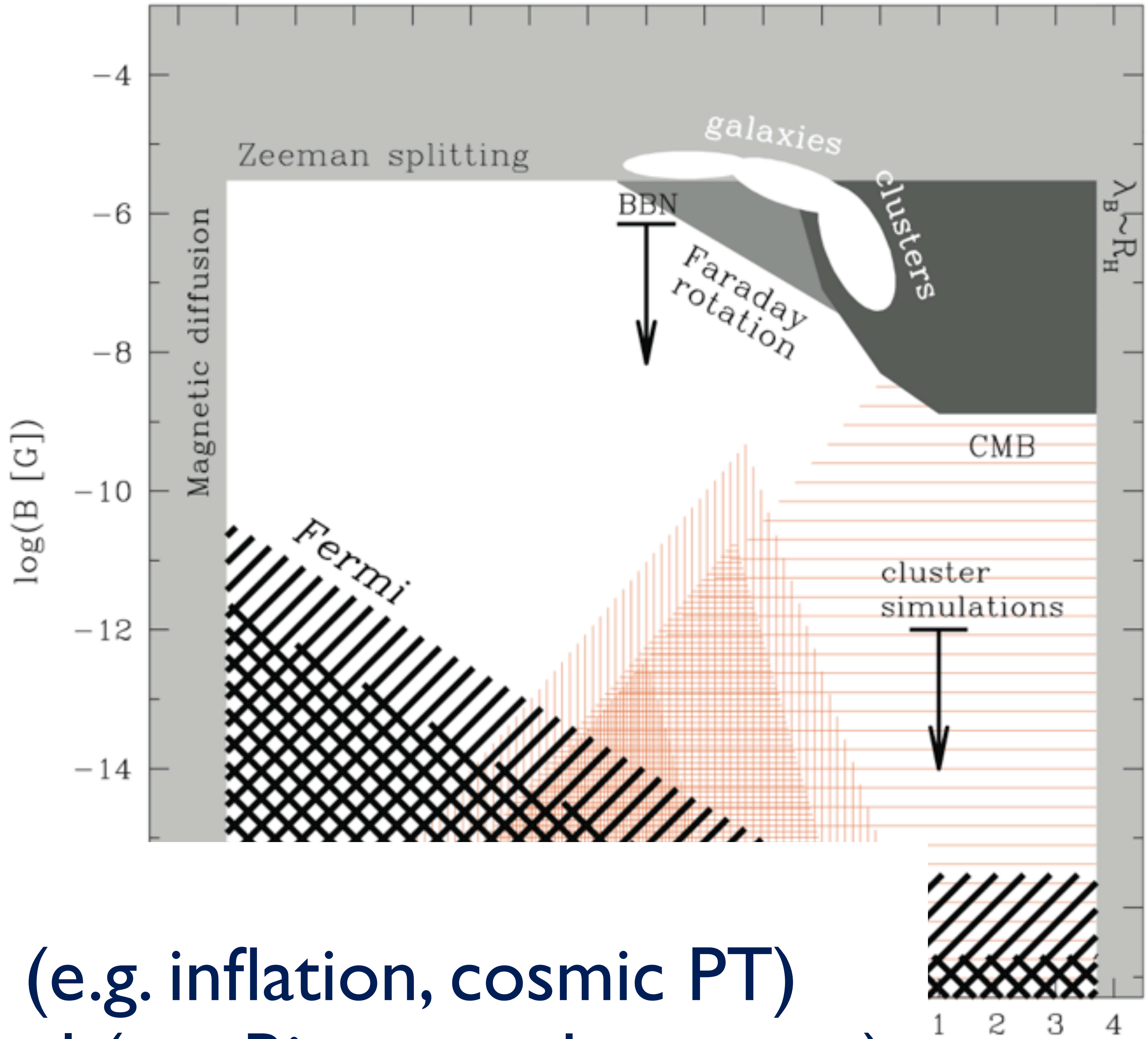

ovk, Science 2010

Origin?

- primordial (e.g. inflation, cosmic PT)
- astrophysical (e.g. Biermann battery, ...)

Generation of Primordial Fields

Possible generation of primordial magnetic fields
(e.g. *Grasso & Rubinstein 2001*; *Widrow 2003*; *Widrow et al. 2011*)

- during cosmic **inflation** (e.g. *Turner & Widrow 1998*)

- during cosmic **phase transitions**
  - electroweak PT ($t \sim 10^{-10}$ sec, $T \sim 100$ GeV)
    (e.g. *Baym et al. 1996*)
  - QCD PT ($T \sim 100$ MeV)

  (e.g. *Quashnock et al. 1989; Cheng & Olinto 1994; Sigl et al. 1997*)

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$\Rightarrow$ causal process
$\Rightarrow$ coherence length limited by Hubble length
at epoch of generation

Evolution of Magnetic Fields

Subsequent evolution

- dilution by cosmic expansion:

$$B \propto a^{-2}$$

assumption: flux freezing (no dynamic damping/ amplification)

- but: damping/amplification is important
(Jedamzik et al. 98, Subramanian & Barrow 98, Banerjee & Jedamzik 2003/2004, Schleicher et al. 2010, Sur et al. 2010)

Evolution of Magnetic Fields

MHD equations on an expanding background:

$$\frac{\partial \rho}{\partial t} + \frac{1}{a} \nabla \cdot \Big((\rho + p)\, \mathbf{v} \Big) + 3\, H\, (\rho + p) = \frac{6}{a} H\, \xi\, (\nabla \cdot \mathbf{v}) - \chi\, \nabla \cdot \mathbf{q}\,,$$

$$\frac{1}{a} \left(\frac{\partial}{\partial t} + \frac{1}{a} (\mathbf{v} \cdot \nabla) + H \right) \mathbf{v} + \frac{1}{a} \frac{\mathbf{v}}{\rho + p} \frac{\partial p}{\partial t} + \frac{1}{a^2} \frac{\nabla p}{\rho + p} + \frac{1}{a^2} \left(\frac{\mathbf{B} \times (\nabla \times \mathbf{B})}{4\pi\, (\rho + p)} \right) =$$
$$\frac{1}{a^3} \frac{\nu}{\rho + p} \left(\nabla^2 \mathbf{v} + \frac{1}{3} \nabla\, (\nabla \cdot \mathbf{v}) \right) + \frac{1}{a^3} \frac{\xi}{\rho + p} \nabla\, (\nabla \cdot \mathbf{v}) - \frac{1}{\rho + p} \left(\frac{\partial}{\partial t} + 5\, H \right) \chi\, \mathbf{q}\,,$$

$$\frac{1}{a} \left(\frac{\partial}{\partial t} + 2\, H \right) \mathbf{B} = \frac{1}{a^2} \nabla \times (\mathbf{v} \times \mathbf{B})$$

- ν, ξ viscosity; χ, $\mathbf{q}$ heat conductivity/heat flux
- H, a Hubble parameter/scale factor $\Longrightarrow$ cosmic expansion

assumption: infinite conductivity $\Longrightarrow$ large Prandtl numbers

Evolution of Magnetic Fields

MHD equations on an expanding background with *super co-moving* variables* (e.g. *Enqvist 98*):

$$\begin{aligned}
\frac{\partial\tilde\rho}{\partial\tilde t} + \nabla\cdot(\tilde\rho\,\tilde{\mathbf{v}}) &= 0 \quad ,\\
\frac{\partial\tilde{\mathbf{v}}}{\partial\tilde t} + (\tilde{\mathbf{v}}\cdot\nabla)\,\tilde{\mathbf{v}} + \frac{1}{\tilde\rho}\nabla\,\tilde p + \frac{1}{4\pi\tilde\rho}\tilde{\mathbf{B}}\times(\nabla\times\tilde{\mathbf{B}}) &= -\tilde{\mathbf{s}} \quad ,\\
\frac{\partial\tilde\epsilon}{\partial\tilde t} + \nabla\cdot(\tilde\epsilon\,\tilde{\mathbf{v}}) + \tilde p\,(\nabla\cdot\tilde{\mathbf{v}}) &= -\tilde\Gamma \quad ,\\
\frac{\partial\tilde{\mathbf{B}}}{\partial\tilde t} - \nabla\times(\tilde{\mathbf{v}}\times\tilde{\mathbf{B}}) &= 0 \quad .
\end{aligned}$$

$\Longrightarrow$ no Hubble-expansion

$\Longrightarrow$ same form than non-relativistic MHD equations

*
$$\begin{array}{lll}
\tilde\rho \equiv \rho\,a^3 & \tilde p \equiv p\,a^4 & \tilde{\mathbf{B}} \equiv \mathbf{B}\,a^2 \\
\tilde{\mathbf{v}} \equiv \mathbf{v}\,a^{1/2} & \tilde\epsilon \equiv \epsilon\,a^4 & \tilde T \equiv T\,a^2 \\
\tilde\chi \equiv \chi\,a^{3/2} & \tilde\nu \equiv \nu\,a^{5/2} & d\tilde t \equiv dt\,a^{-3/2} \\
\tilde\xi \equiv \xi\,a^{5/2} & \tilde H \equiv a^{3/2}\,H &
\end{array}$$

Evolution of Magnetic Fields

Further assumptions:

- incompressible MHD:

  $v,\ v_A \ll c_s$

  $v_A < c_s$ if $B < 5\text{x}10^{-5}$ G at recombination

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} - (\mathbf{v}_A\cdot\nabla)\mathbf{v}_A = \mathbf{f},$$

$$\frac{\partial \mathbf{v}_A}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}_A - (\mathbf{v}_A\cdot\nabla)\mathbf{v} = \nu\nabla^2\mathbf{v}_A,$$

dissipation:

$$\mathbf{f} = \begin{cases} \eta\nabla^2\mathbf{v} & l_{\rm mfp} \ll l, \\ -\alpha\mathbf{v} & l_{\rm mfp} \gg l, \end{cases}$$

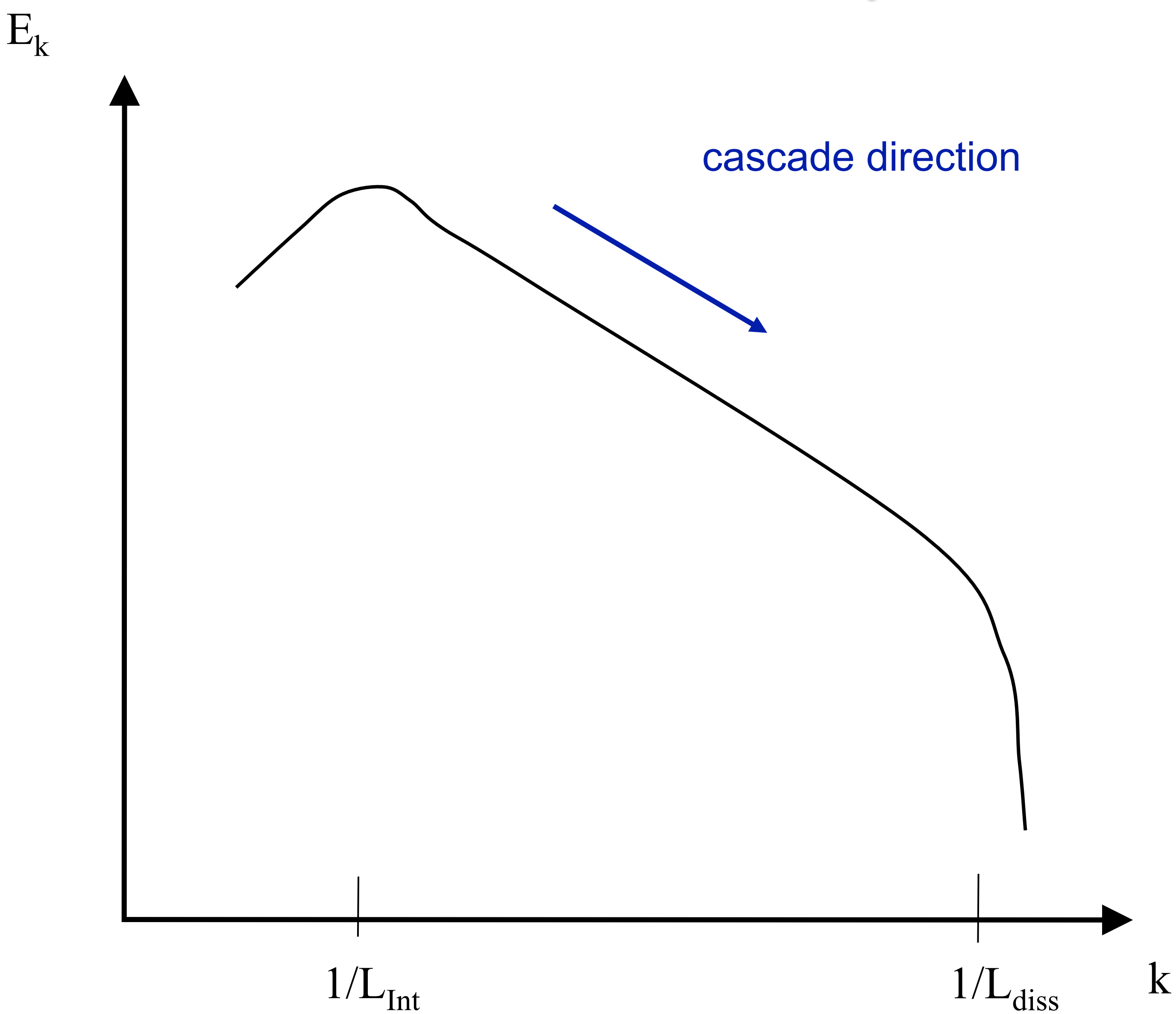
Reynolds number:

$$R_e(l) = \frac{v^2/l}{|\mathbf{f}|} = \begin{cases} \frac{vl}{\eta} & l_{\rm mfp} \ll l \\ \frac{v}{\alpha l} & l_{\rm mfp} \gg l, \end{cases}$$

Evolution of Magnetic Fields

$$\mathrm{R_e} \gg 1 \quad \Longrightarrow \quad \text{decay via MHD turbulence}$$

$$E \approx \int d\ln k k^3 (\langle |\boldsymbol{v}_k|^2 \rangle + \langle |\boldsymbol{v}_{\mathrm{A},k}|^2 \rangle) \equiv \int d\ln k E_l,$$



quasi-stationary transfer of energy in *k*-space

$\Longrightarrow$ Kolmogorov Turbulence

$$\frac{\mathrm{d}E_l}{\mathrm{d}t} \approx \frac{E_l}{\tau_l} \approx \mathrm{const}(l)$$

Evolution of Magnetic Fields

turbulent decay laws

- Initial spectrum on large scales ($l > \mathrm{L}$):

$$E_k \approx E_0\left(\frac{k}{k_0}\right)^n = E_0\left(\frac{l}{L_0}\right)^{-n} \quad \text{for } l > L_0.$$

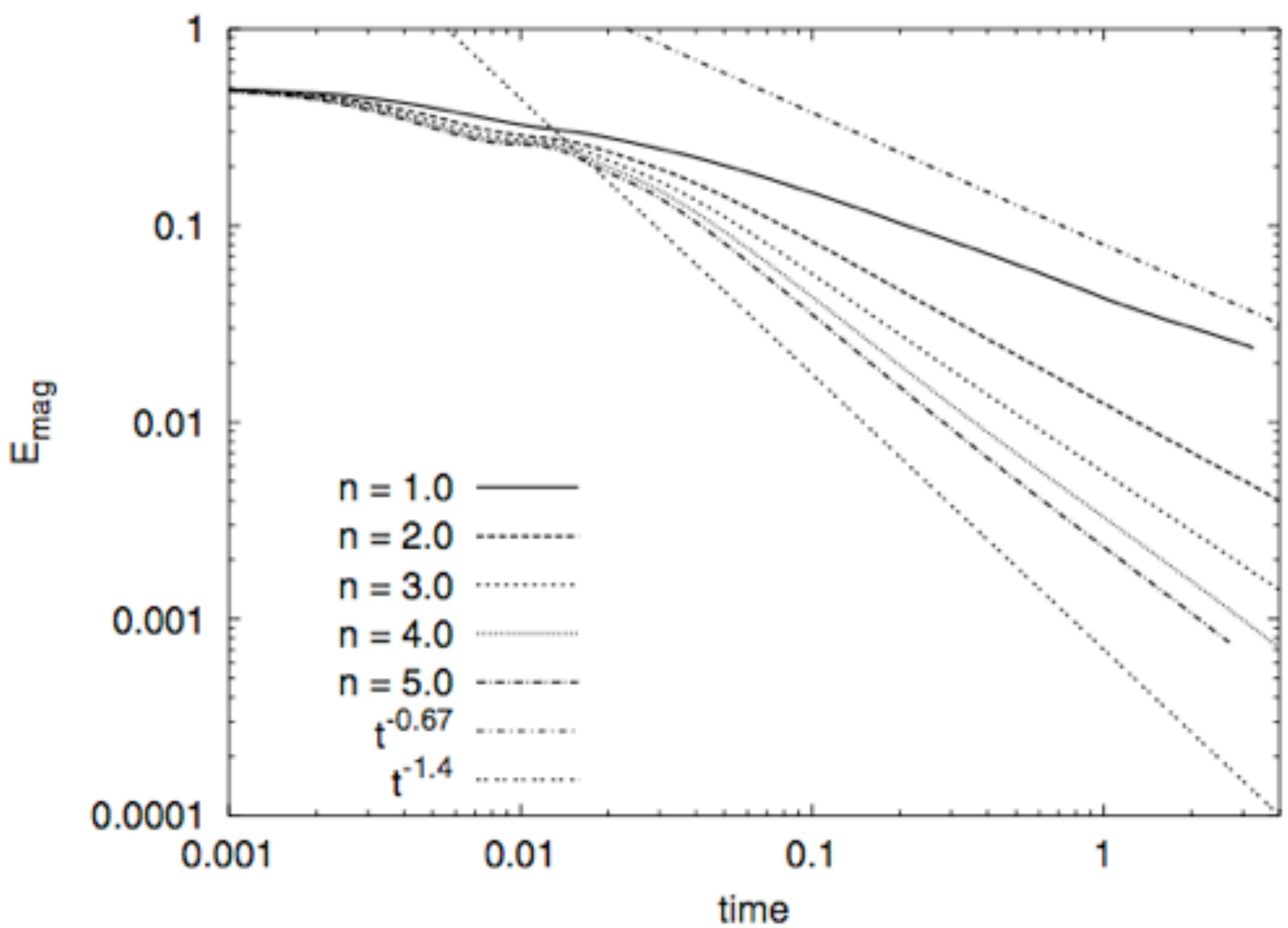
- with: $v_l = \sqrt{E_l}$:

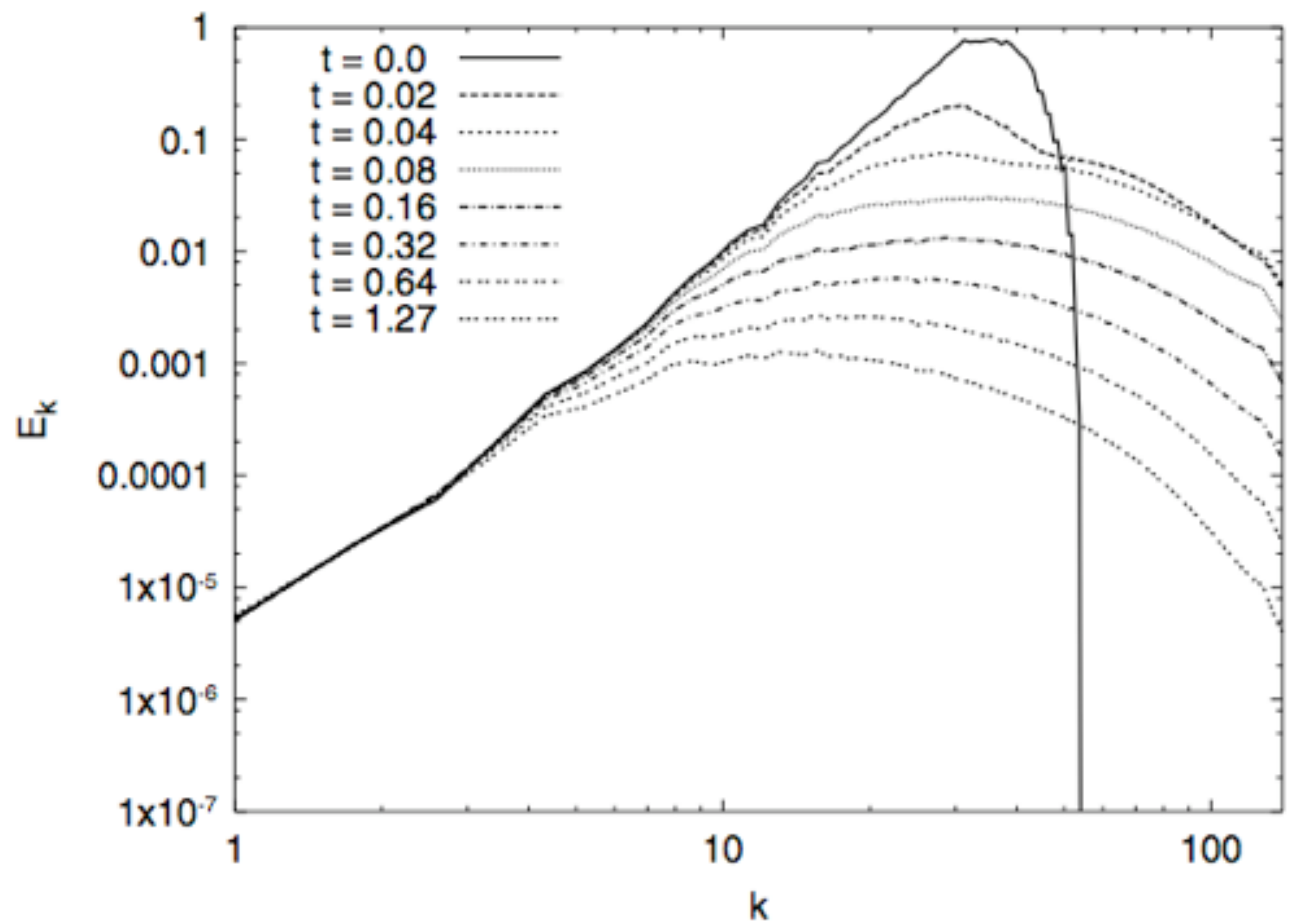
energy decay: $$E \approx E_0\left(\frac{t}{\tau_0}\right)^{-2n/(2+n)}$$

increase of coherence length: $$L \approx L_0\left(\frac{t}{\tau_0}\right)^{2/(2+n)}$$

Evolution of Magnetic Fields

turbulent decay laws





- decay law: $$E \approx E_0 \left(\frac{t}{\tau_0} \right)^{-2n/(2+n)}$$
- growths of coherence length: $$L \approx L_0 \left(\frac{t}{\tau_0} \right)^{2/(2+n)}$$

Helicity

Helical Fields

- Helicity (measures complexity of the field):

$$\mathcal{H} \equiv \frac{1}{V} \int_V \mathrm{d}^3x \, \mathbf{A} \cdot \mathbf{B}$$

is conserved (no resistivity)

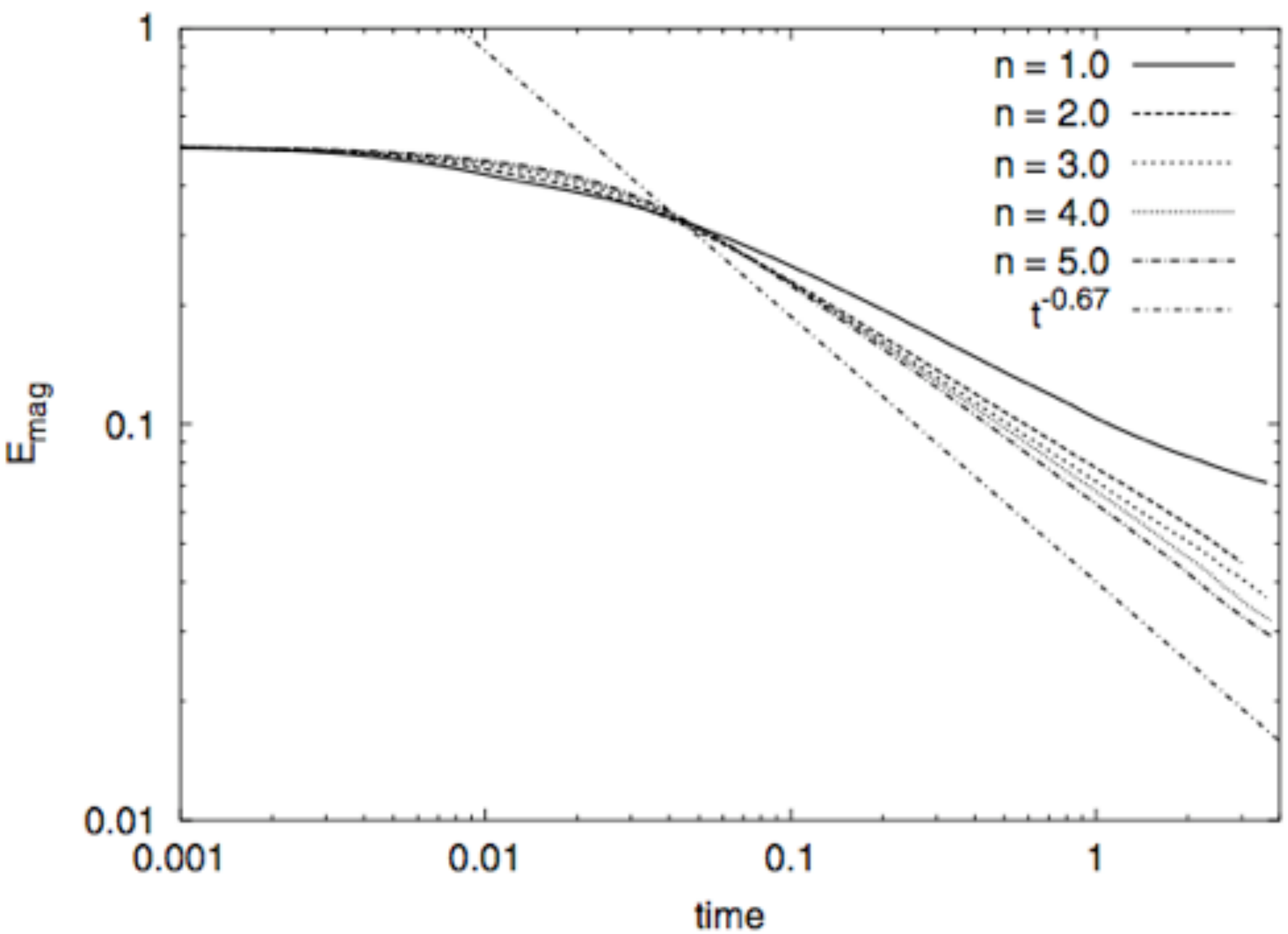
- maximal helical field: $H \sim B^2 L \approx E L$

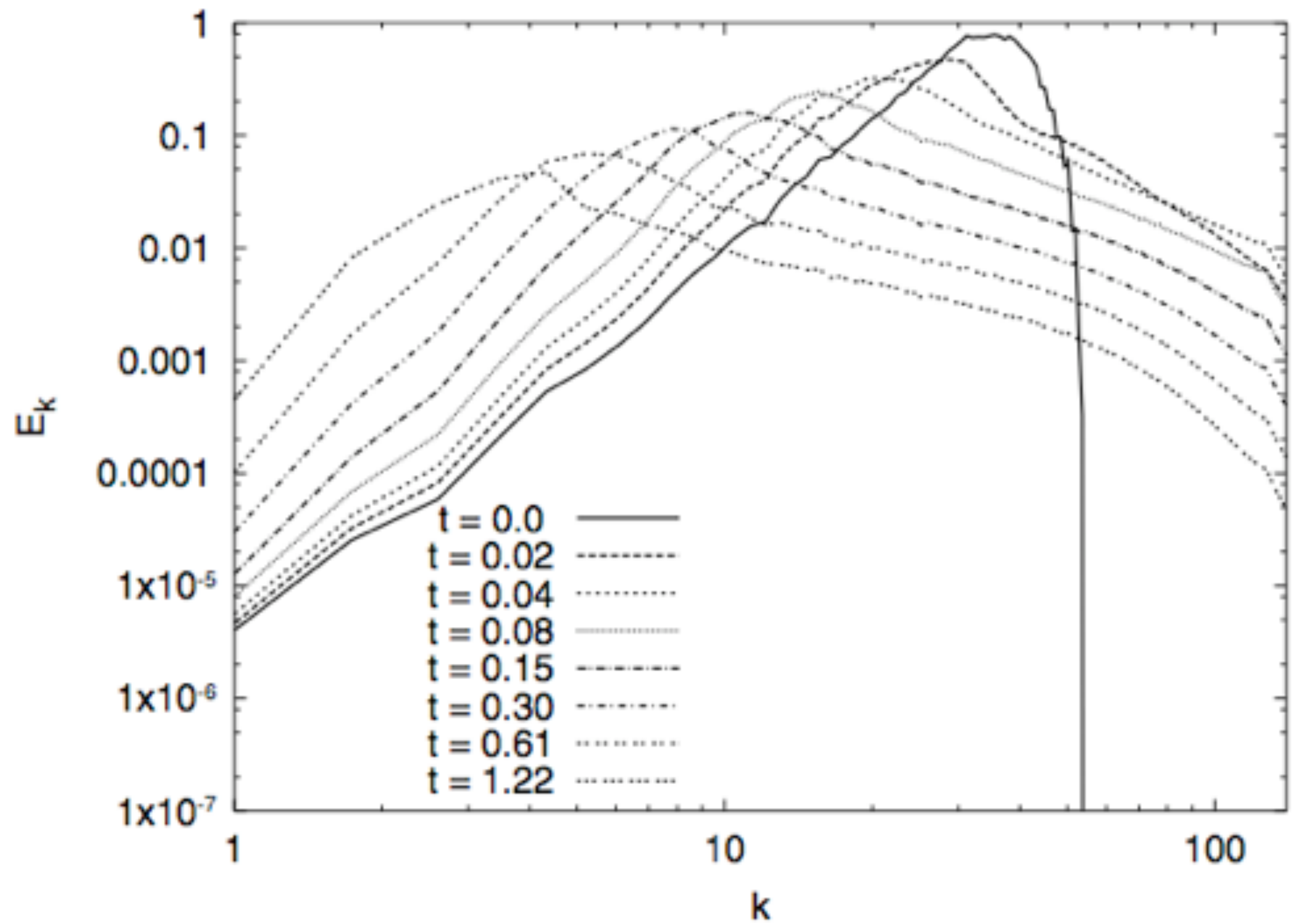
energy decay: $$E \approx E_0 \left(\frac{t}{\tau_0} \right)^{-2/3}$$

inverse cascade: $$L \approx L_0 \left(\frac{t}{\tau_0} \right)^{2/3}$$

Evolution of Magnetic Fields

- Fields with maximum helicity: $\mathcal{H}_{\max} \approx \langle B^2 L \rangle \approx (8\pi) E L$



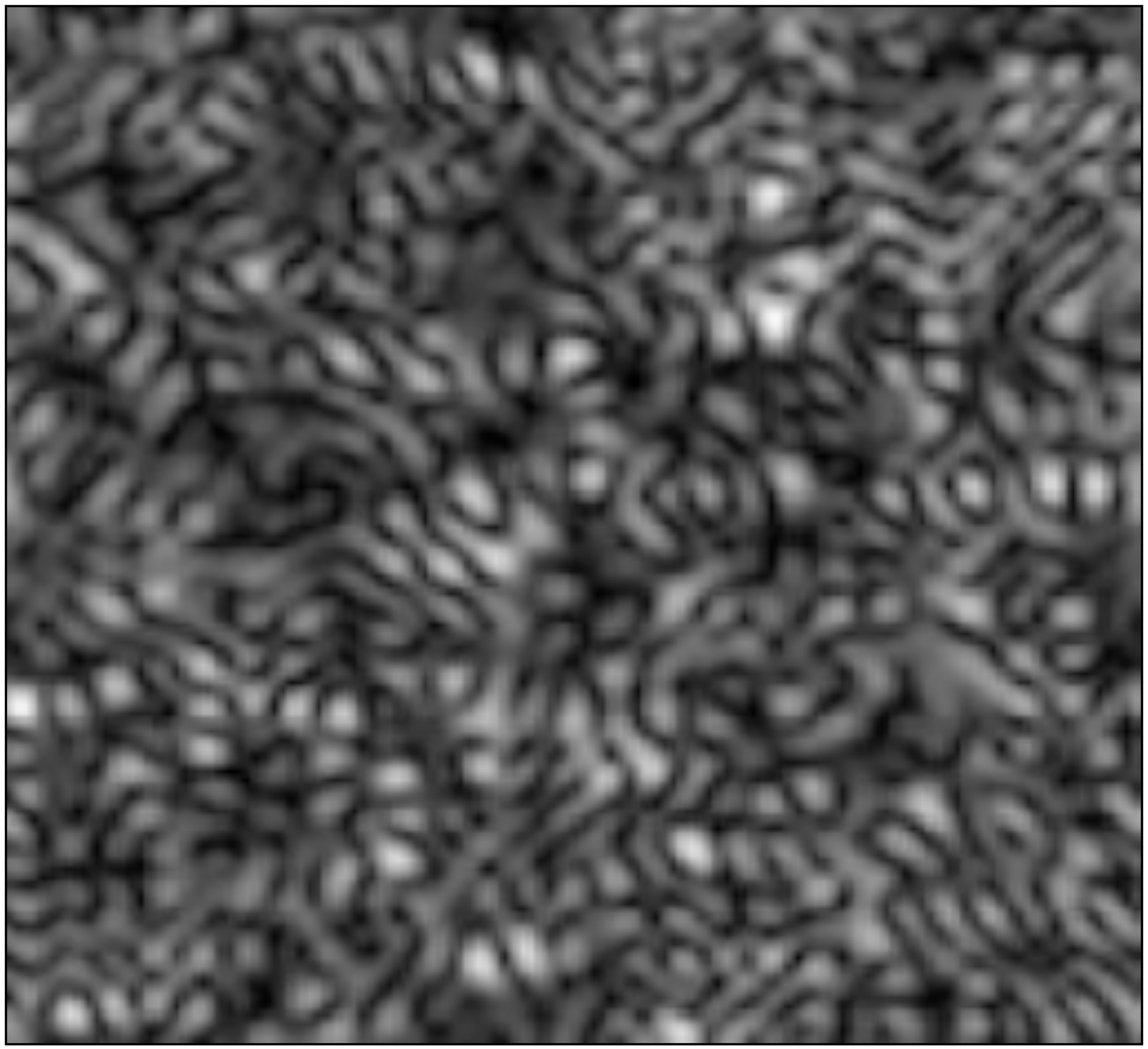


- decay law: $E \approx E_0 \left(\frac{t}{\tau_0}\right)^{-2/3}$

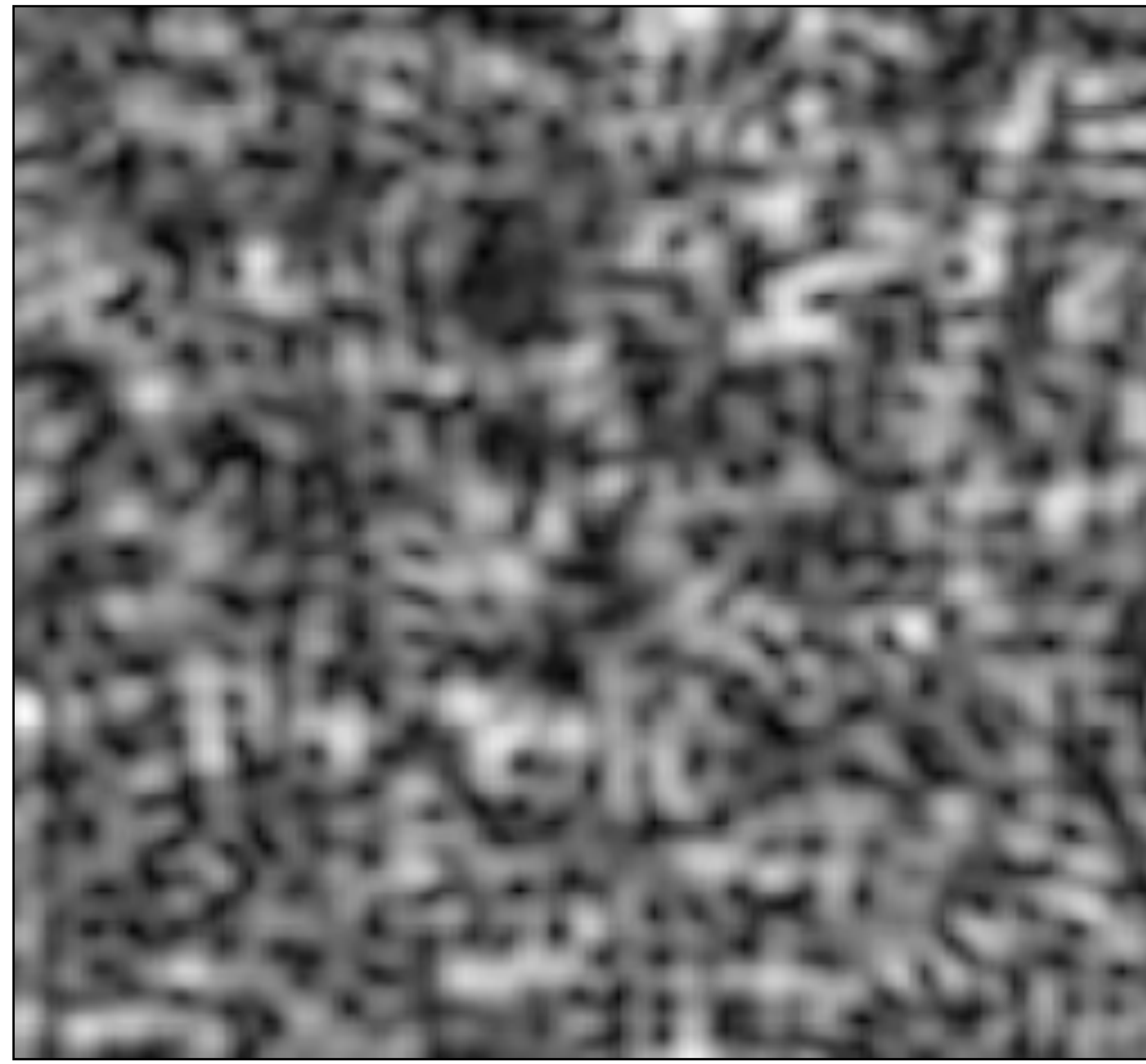
- growths of coherence length: $L \approx L_0 \left(\frac{t}{\tau_0}\right)^{2/3}$ inverse cascade

Evolution of Magnetic Fields

Evolution of small scale random magnetic fields



no initial helicity



with max. initial helicity

Evolution of Magnetic Fields

Viscous regime ($R_e < 1$):

$$v_L(T) = \frac{v^2_{A,L}(T)\,L}{\eta(T)}$$

for $l_{mfp} \ll L$

$$v_L(T) = \frac{v^2_{A,L}(T)}{\alpha(T)\,L}$$

for $l_{mfp} \gg L$

$$\Longrightarrow \qquad \tau_{\rm visc.damp} = \begin{cases} \dfrac{\nu}{V_A^2} \\ \dfrac{\alpha\,L^2}{V_A^2} \end{cases}$$

$\Longrightarrow$ overdamped modes for $t < \tau_{\rm visc}$

(Jedamzik et al. 1998, RB & Jedamzik 2004)

$\Longrightarrow$ $B(t) \approx const$

Evolution of primordial fields

apply to cosmic evolution

$\Longrightarrow$ evolution equation:

$$\tau_L \approx \frac{L(T)}{v_L(T)} \approx \frac{1}{H(T)} \approx t_{\mathrm{H}}$$

turbulent regime ($\mathrm{R_e} \gg 1$): $v_L(T) = v_{\mathrm{A},L}(T)$

viscous regime ($\mathrm{R_e} < 1$):

$$v_L(T) = \frac{v_{\mathrm{A},L}^2(T)\, L}{\eta(T)}$$

for $l_{\mathrm{mfp}} \ll L$

$$v_L(T) = \frac{v_{\mathrm{A},L}^2(T)}{\alpha(T)\, L}$$

for $l_{\mathrm{mfp}} \gg L$

Evolution of primordial fields

viscosity η and drag α

- **neutrinos**:

$$\eta_\nu \propto l_{\mathrm{mfp},\nu} \propto T^{-5}$$
$$\alpha_\nu \propto 1/l_{\mathrm{mfp},\nu} \propto T^{5}$$

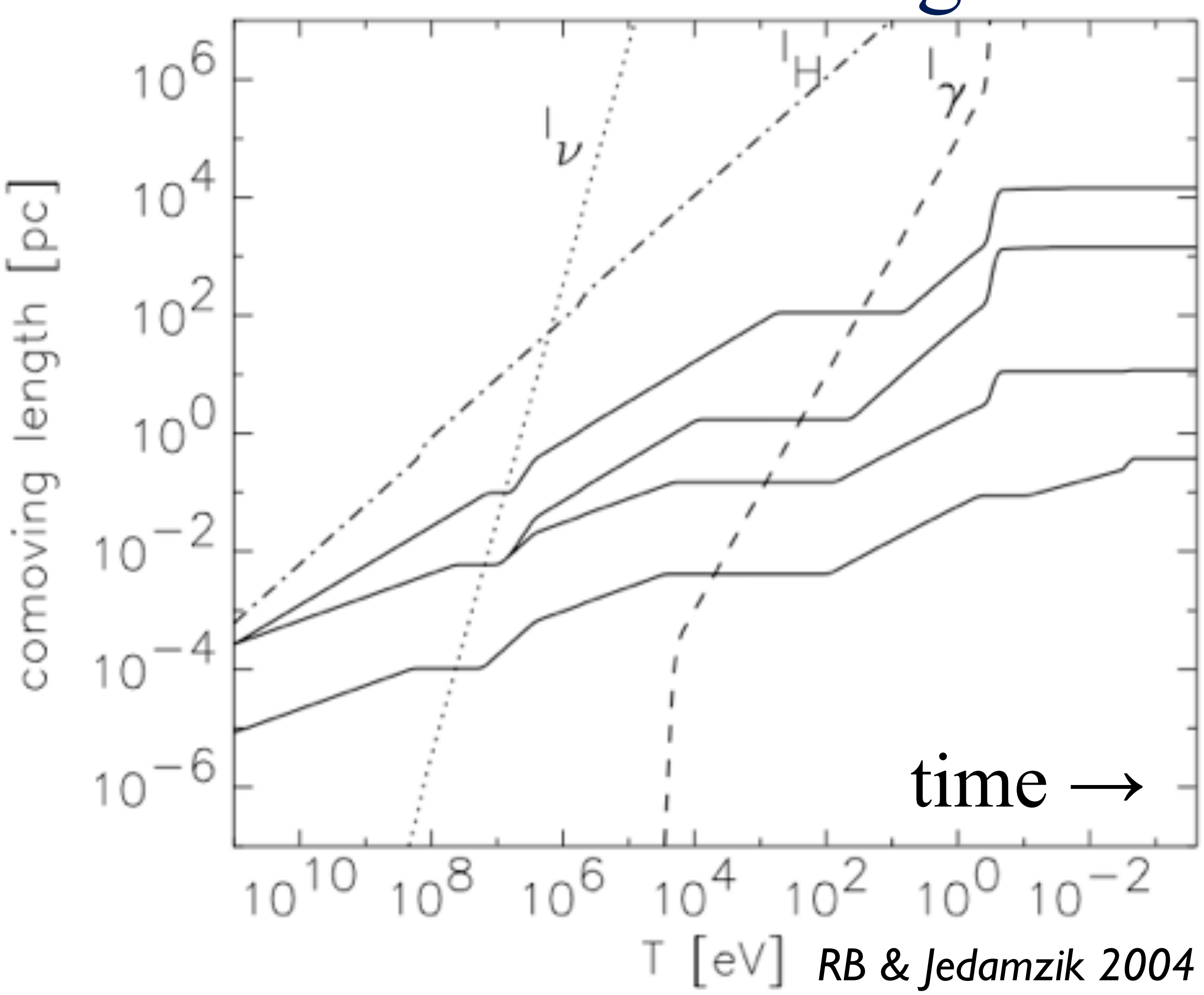
- **photons**:

$$\eta_\gamma \propto l_{\mathrm{mfp},\gamma} \propto T^{-3}$$
$$\alpha_\gamma \propto \rho_\gamma \propto T^{-4}$$

Evolution of primordial fields

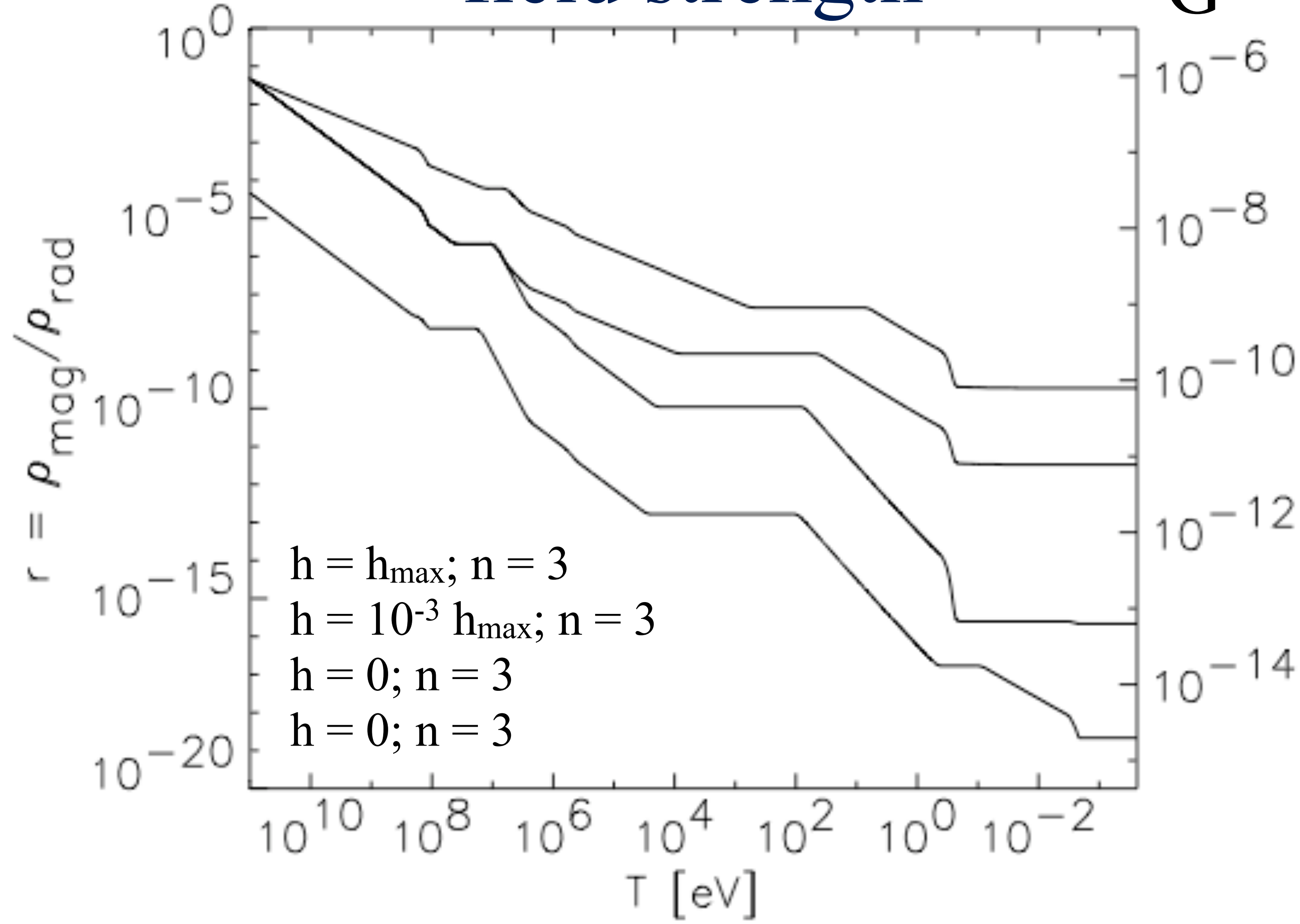
combine with cosmic evolution

coherence length



RB & Jedamzik 2004

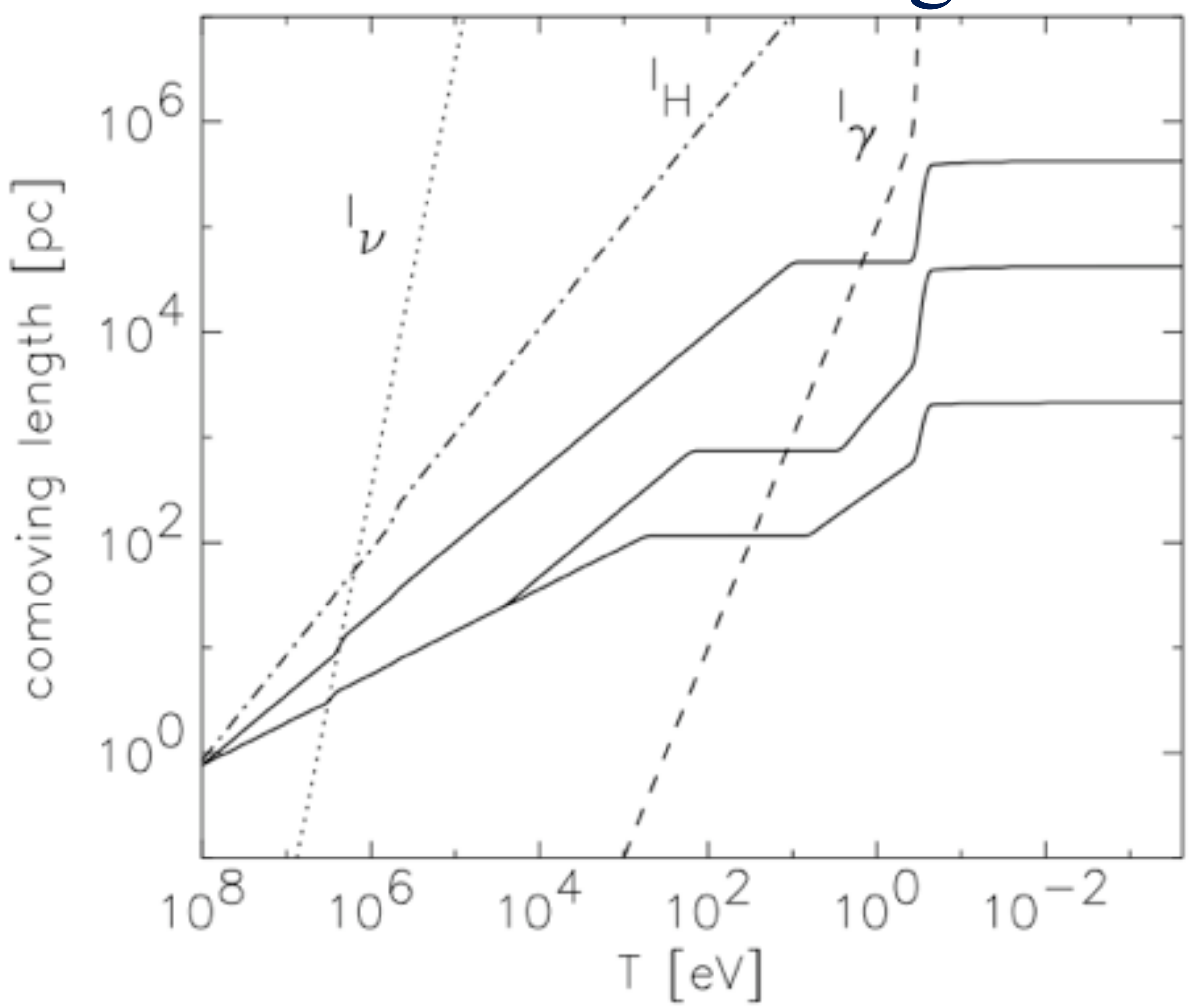
field strength

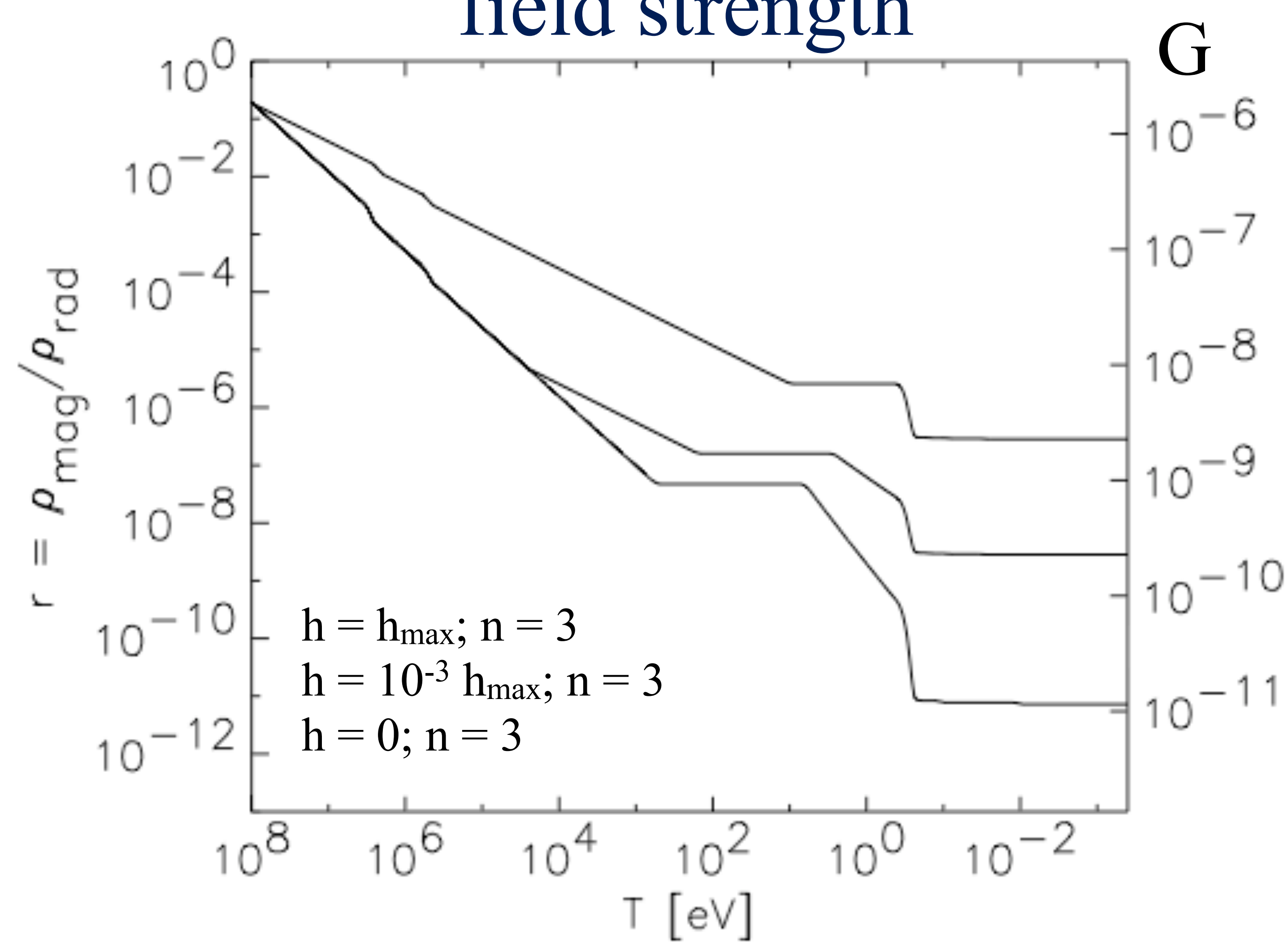


assume magneto-genesis at EW-PT ($T_{gen} = 100$ GeV)

Evolution of primordial fields

combine with cosmic evolution



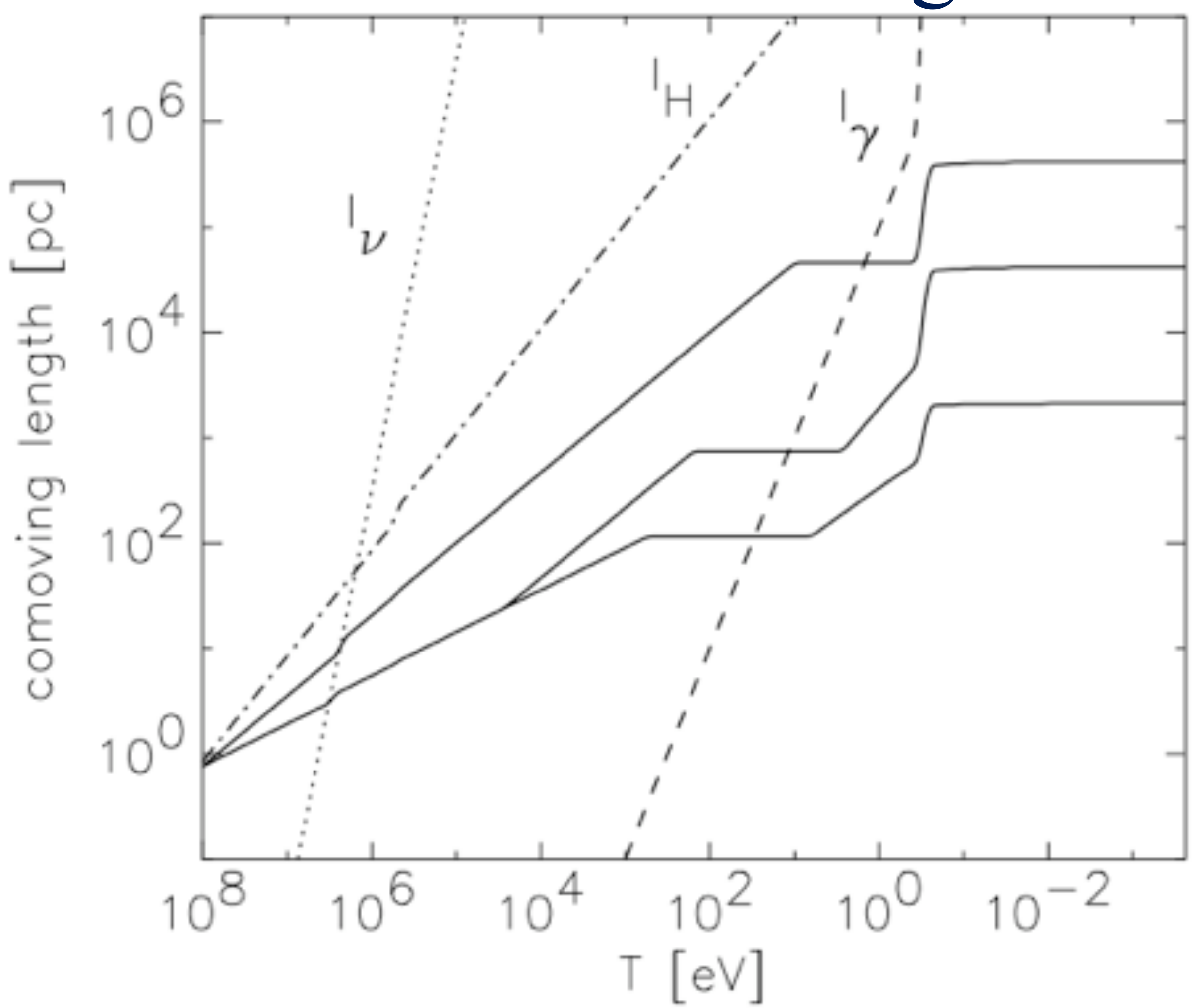


assume magneto-genesis at QCD-PT ($T_{gen} = 100$ MeV)

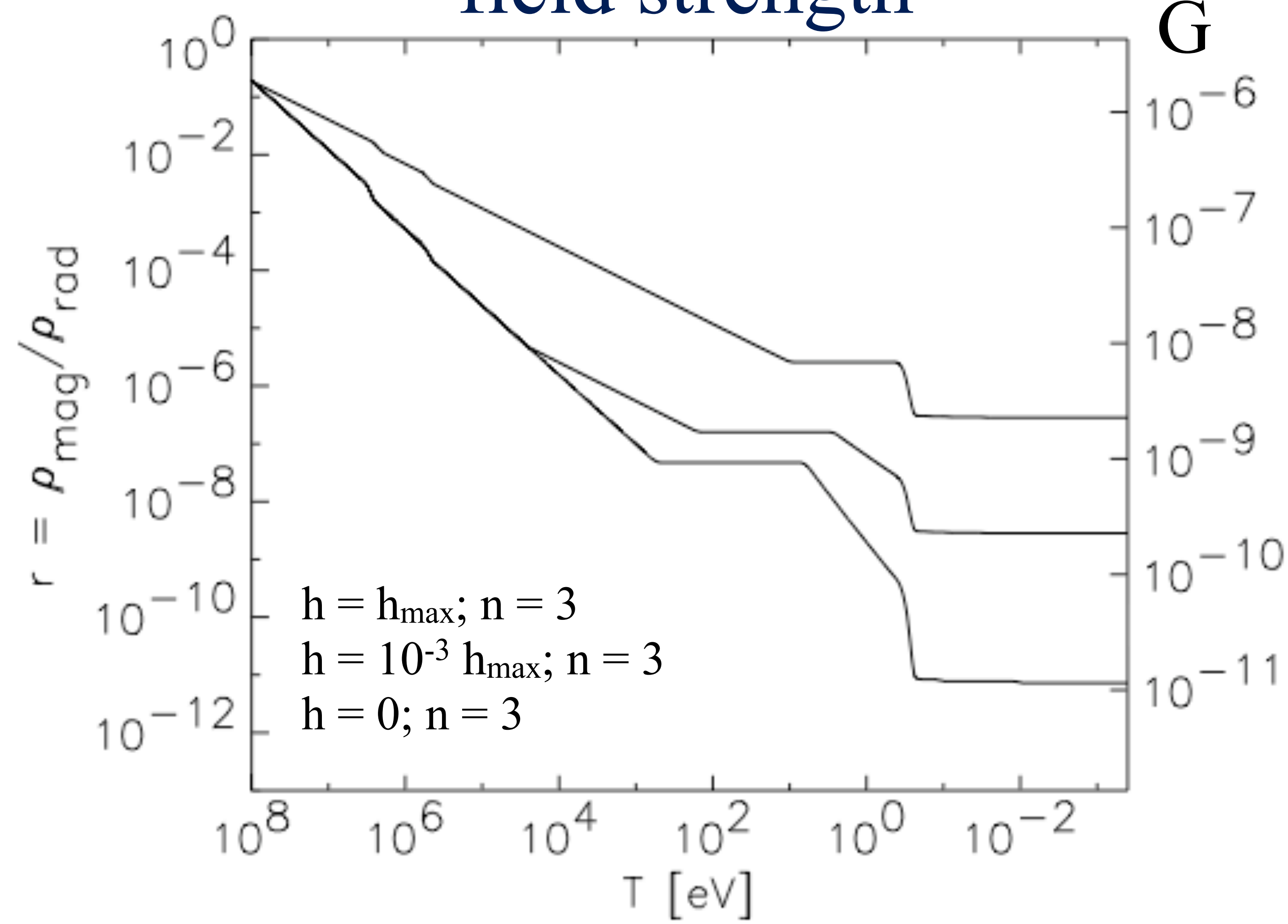
Evolution of primordial fields

combine with cosmic evolution

coherence length



field strength



assume magneto-genesis at QCD-PT ($T_{gen} = 100$ MeV)

Cluster fields of primordial origin?

Evolution of primordial fields

Cluster fields of primordial origin?

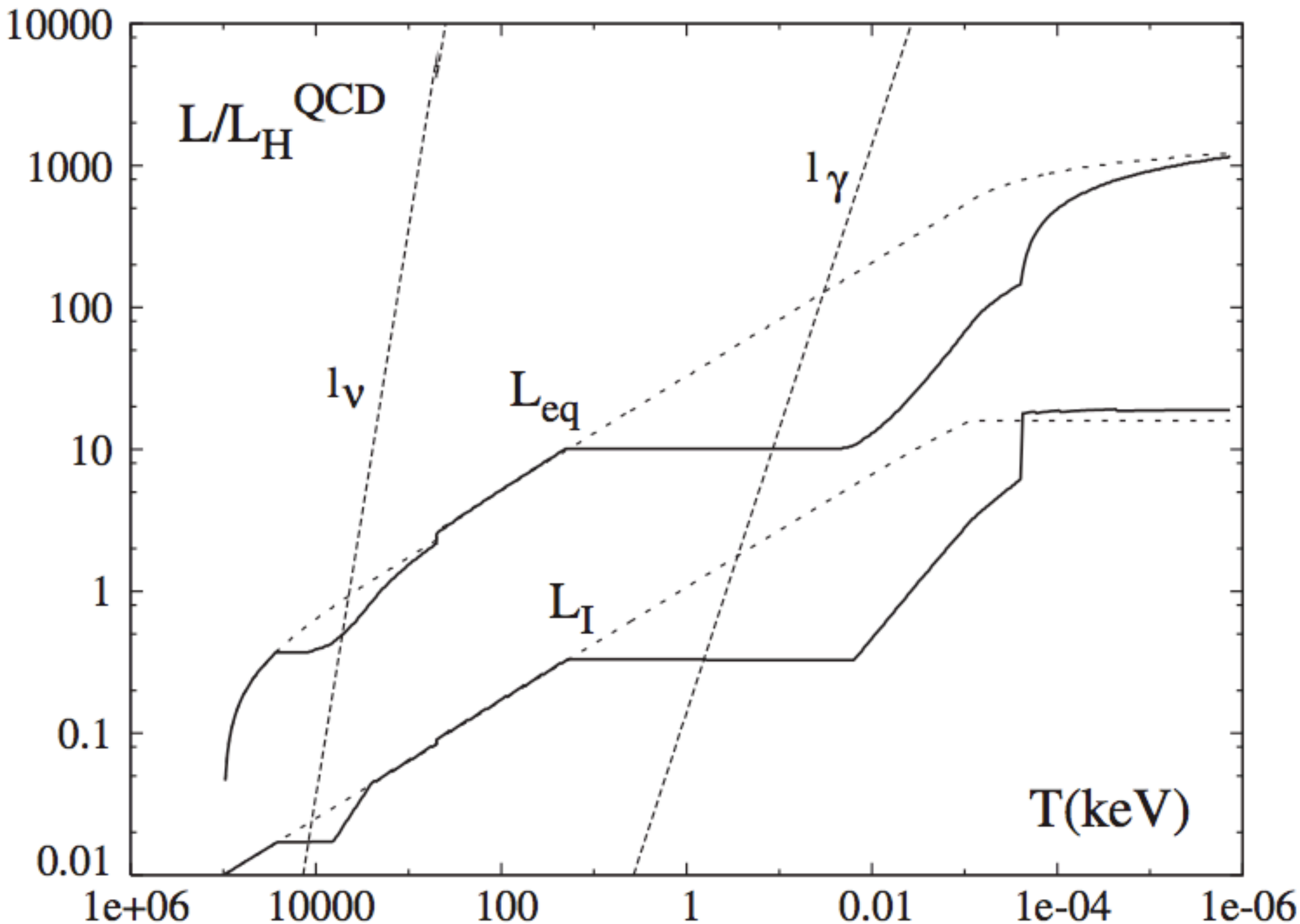
- seed fields:
  - $l_{\mathrm{coh}} \sim 10^5$ pc
  - $B \sim \mathrm{nG}$

- further evolution:
  - amplification by adiabatic compression
  - + turbulent dynamo

  $\Longrightarrow$ μG fields with right structure possible

  (e.g. *Dolag et al. 2002*; *Donnert et al. 2009*;
  see also talks by Dominik Schleicher and Marcus Brüggen)

Evolution of primordial fields

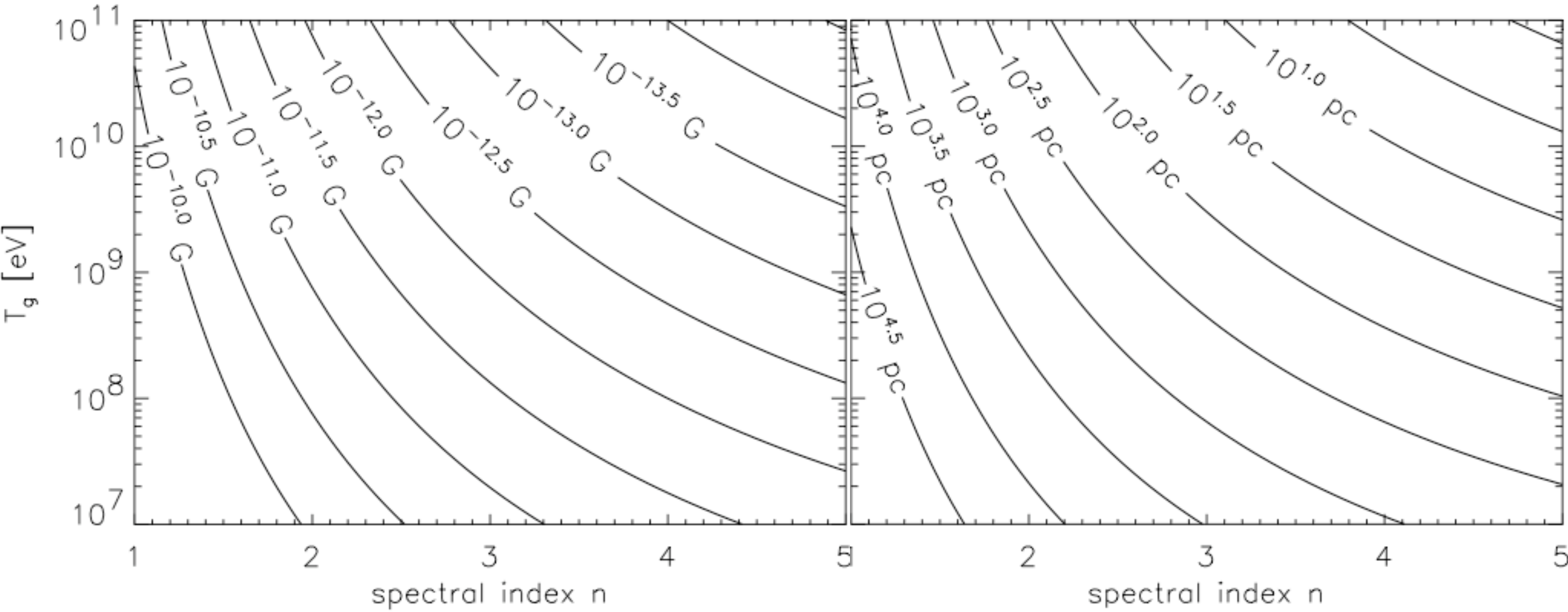


$L_H^{QCD} \approx 1$ pc

Jedamzik & Sigl PRD (2011)

Evolution of primordial fields

combine with cosmic evolution



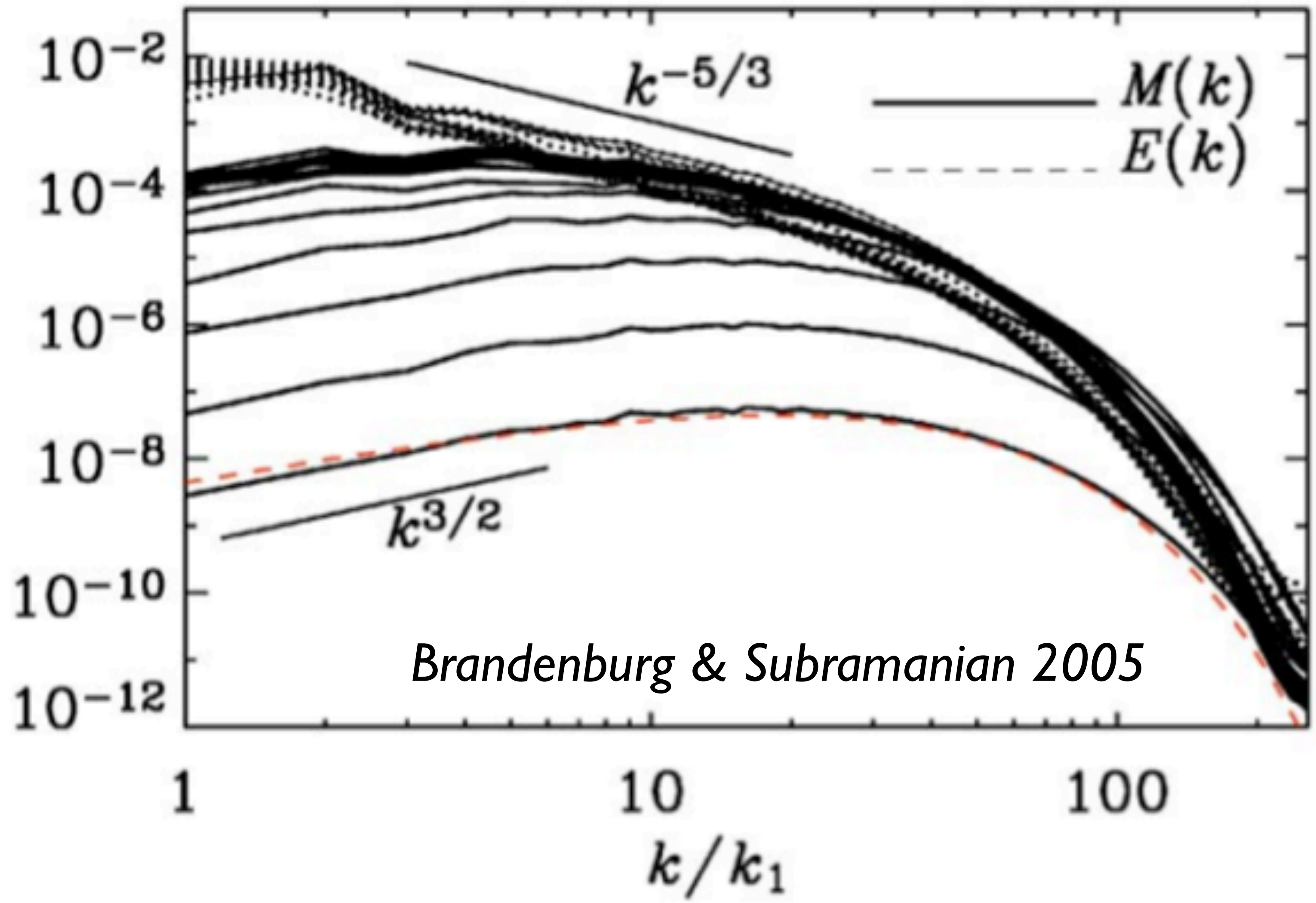
present day field strength and coherence length

B-Field amplification

Small-scale dynamo

(Batchelor 1950, Kazantsev 1968, see also Brandenburg & Subramanian 2005, Schober et al. 2012a,b)

- exponential growth of weak seed fields
- growth rate depends on magnetic Reynolds number: $\gamma \propto \mathrm{Rm}^{-1/2}$
- mag. spectrum: $E_{\mathrm{mag},k} \propto k^{3/2}$
- saturation at $E_{\mathrm{mag}} \sim 0.1\ E_{\mathrm{kin}}$



Brandenburg & Subramanian 2005

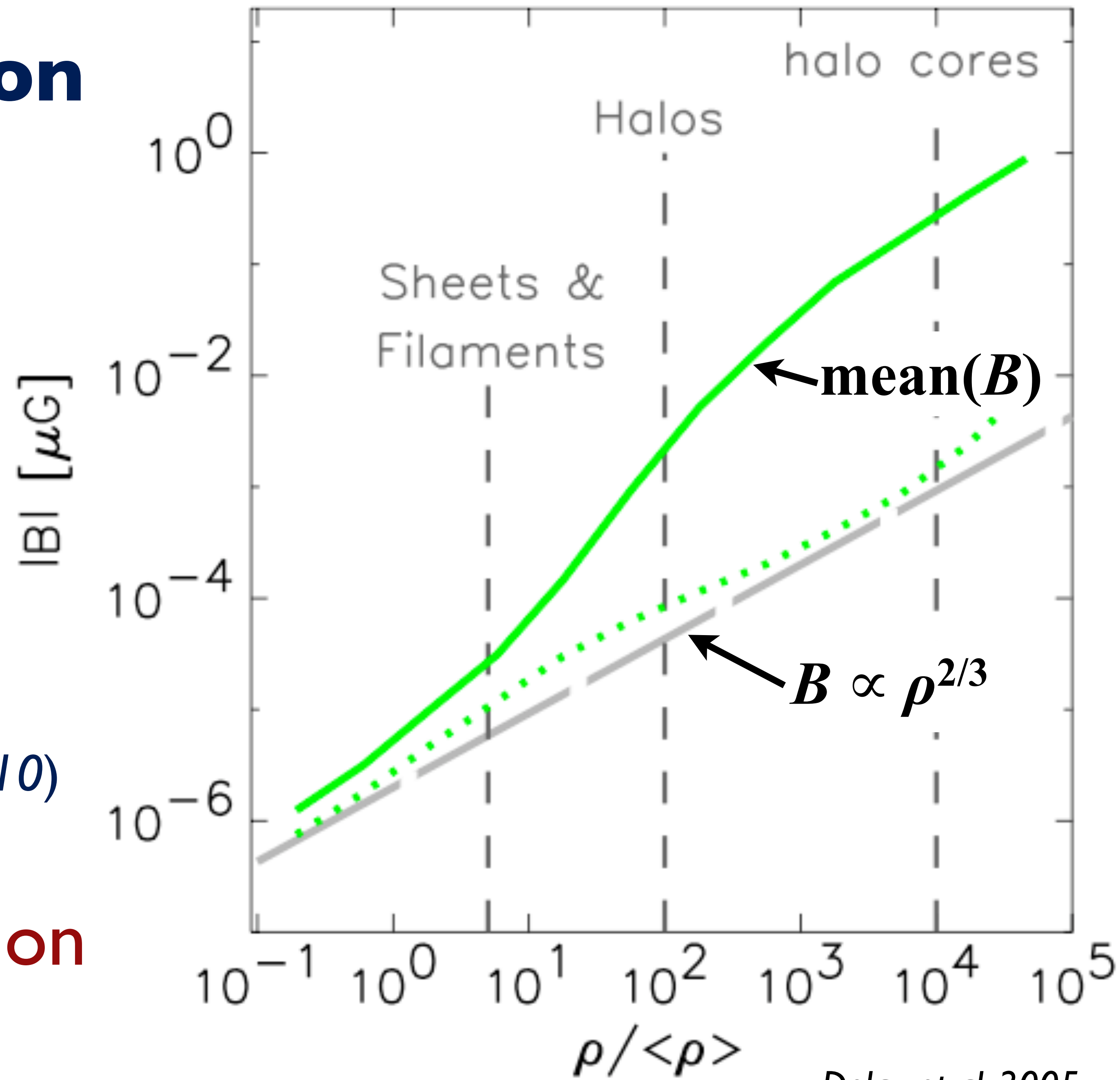
B-Field amplification

B-fields during compression

- maximum growth by adiabatic compression:
  $B \propto \rho^{2/3}$

- small-scale dynamo works in cluster forming models
  (e.g. *Dolag et al. 1999, 2000; Xu et al. 2009, 2010*)

- depends on numerical resolution

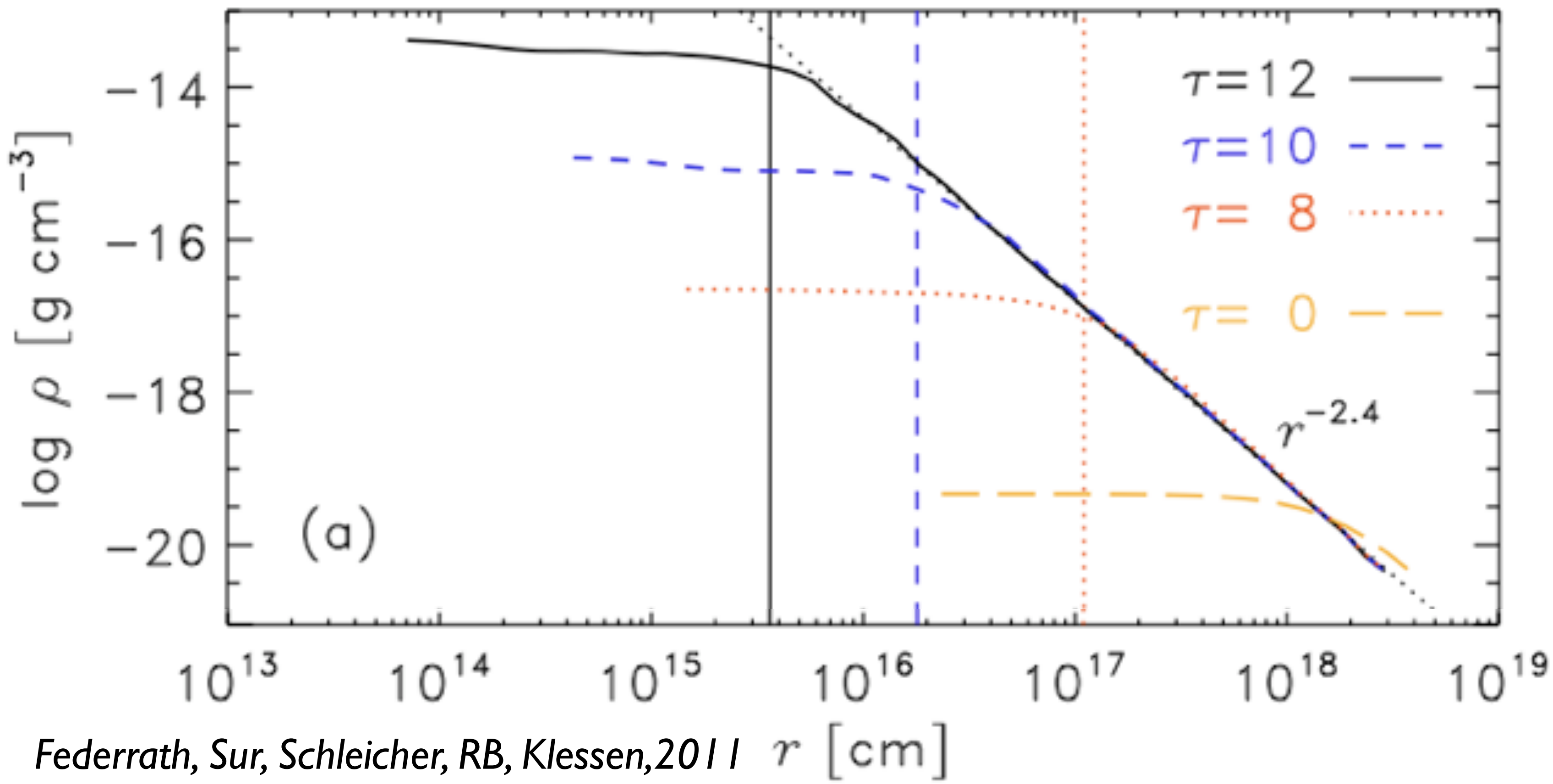


Dolag et al. 2005

Dynamo during "First Star Formation"

- turbulent infall motions (e.g. *Abel et al. 2002, Greif et al. 2008*)

- baryonic core modelled on a supercritical hydrostatic sphere:
  - $M_{\rm baryon} = 1500\ \rm M_{sol}$
  - $\rho_0 = 5\times10^{-20}\ \rm g\ cm^{-3}$
  - weak random field: $B = 1\rm nG$, $\beta = 10^{10}$
  - transonic turbulence: $v_{\rm rms} = 1.1\ \rm km\ sec^{-1}$



Federrath, Sur, Schleicher, RB, Klessen, 2011

characteristic length: **Jeans length**: $\lambda_J = \left(\frac{\pi\, c_s^2}{G\,\rho}\right)^{1/2}$

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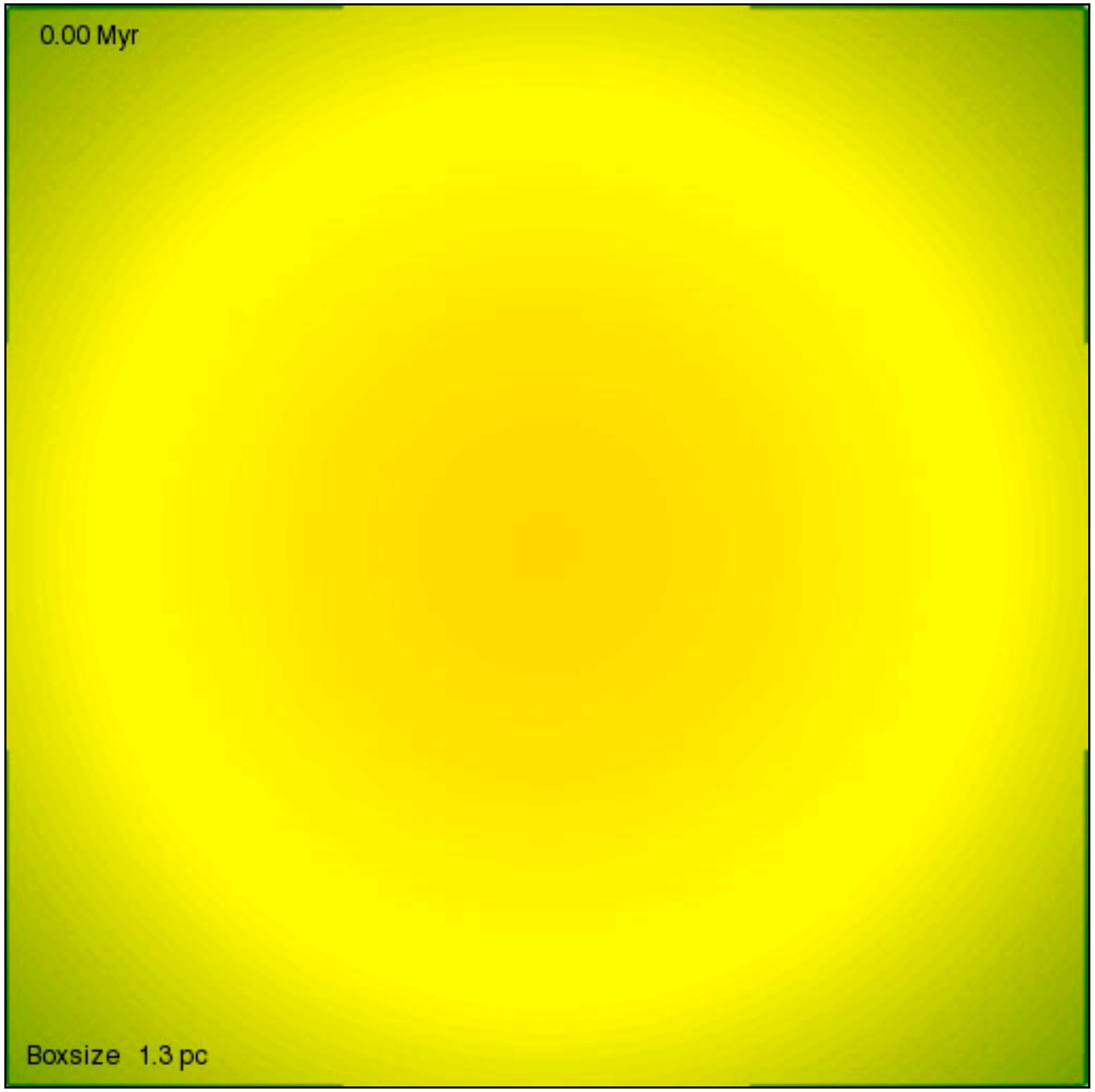

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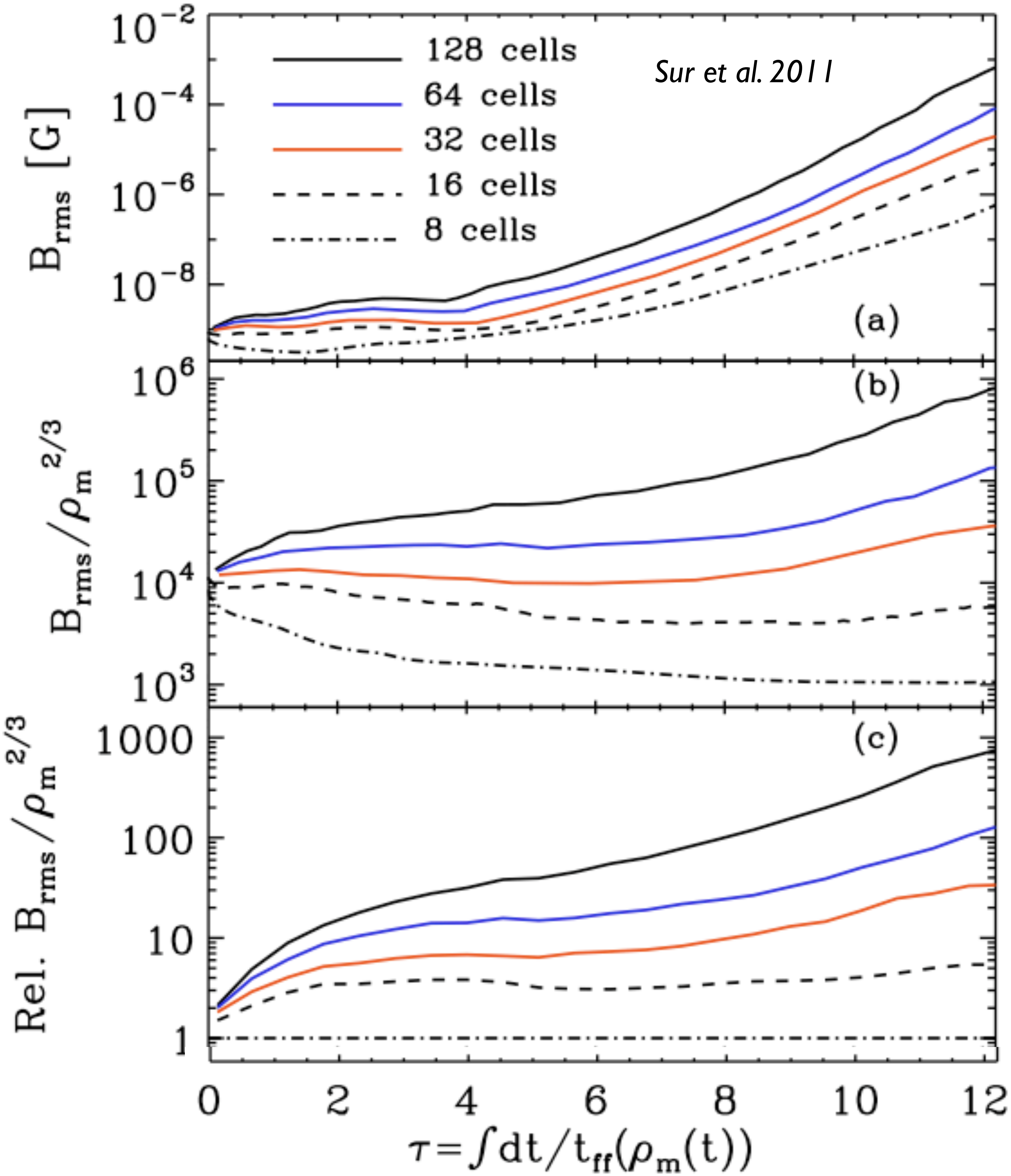
Dynamo during "First Star Formation"



- growth rate depends on R_m, i.e. resolution:

$$R_m \propto N_J^{4/3}$$ (e.g. *Haugen et al. 2004*)

N_J: number of grid cells per local Jeans length; realization with adaptive mesh refinement (AMR)

- minimum resolution: ~ 30 grid cells per Jeans length

Summary

- **Primordial Magnetic Fields** undergo strong dynamic evolution (not only $B \propto a^{-2}$)

- damping in the turbulent regime where $v_A \sim v$
- frozen-in in the viscous regime

- turbulent dynamo: efficient amplification of weak fields (see also talk by Dominik Schleicher)

- Cluster/Galactic magnetic fields are of primordial origin?