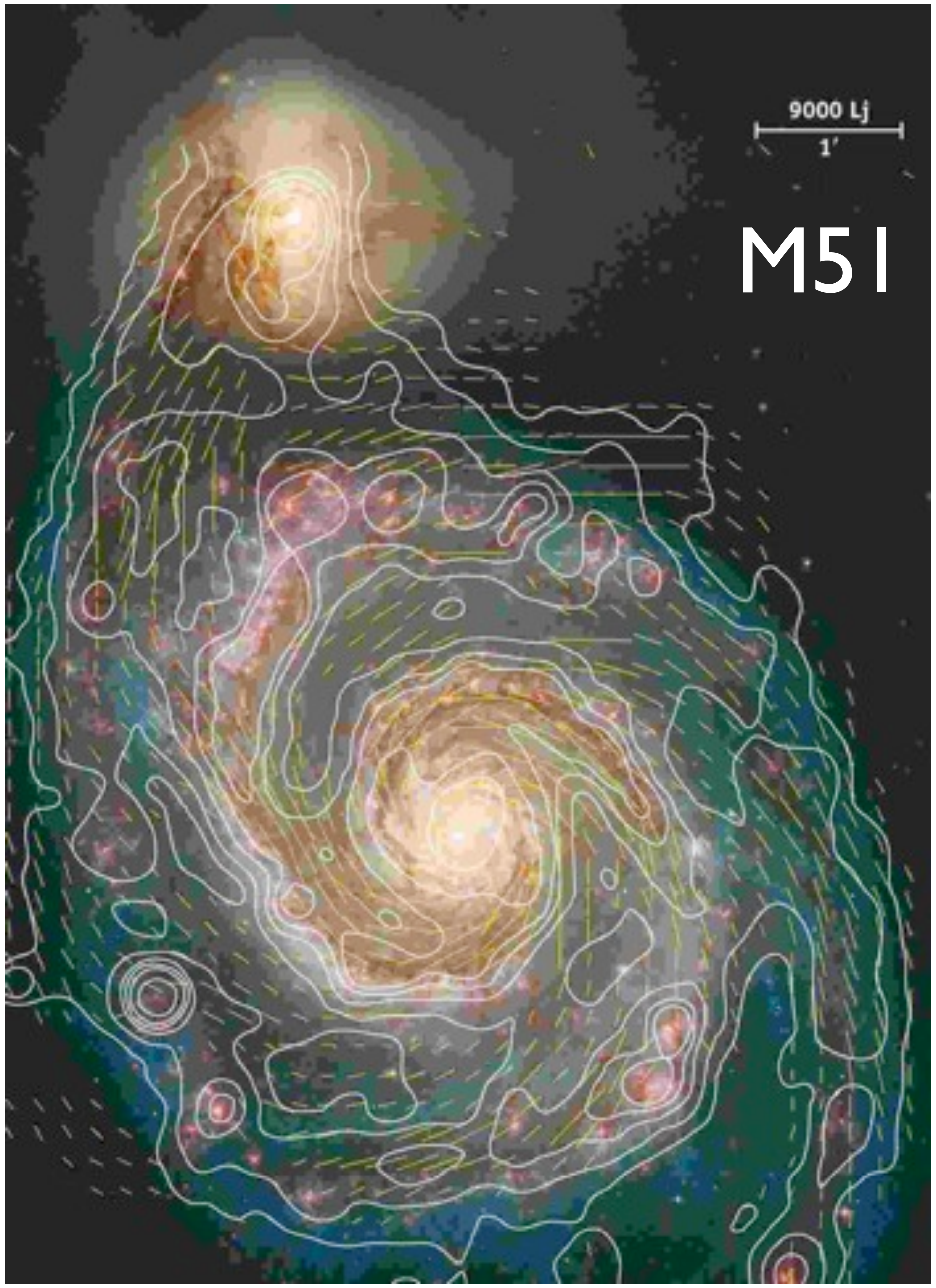
hamburger sternwarte

Evolution and Impact of Cosmic Magnetic Fields

Robi Banerjee
University of Hamburg

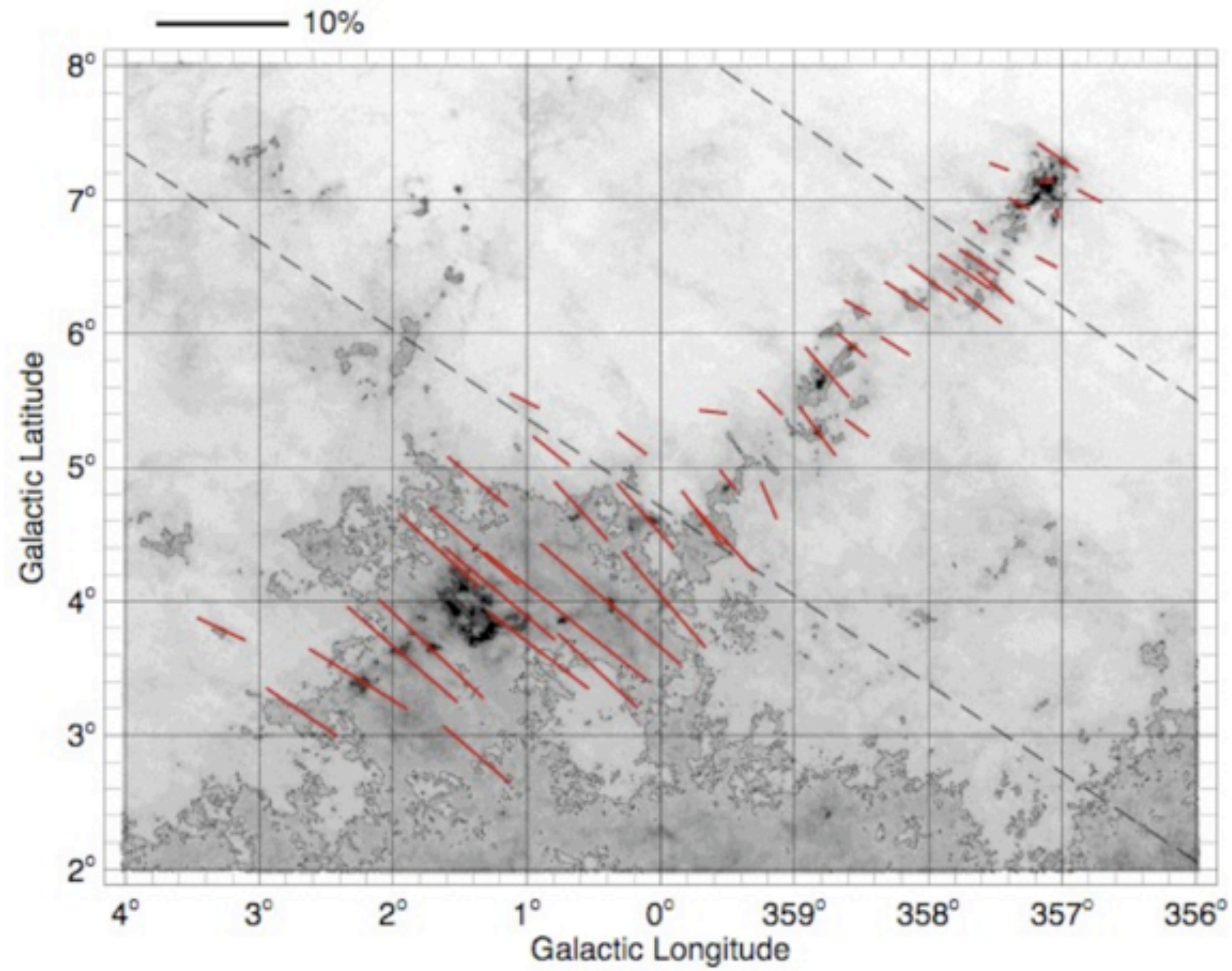
Collaborators:
Dominik Schleicher (Göttingen), C. Federrath (Lyon, HD), Karsten Jedamzik (Montpellier), R. Klessen (Heidelberg), J. Schober (Heidelberg), S. Sur (Heidelberg)

Observed Magnetic Fields



galactic B-fields (e.g. *R.Beck 2001*)
large scale component: ~ 4μG
total field strength: ~ 6μG

Magnetic fields are observed on all scales

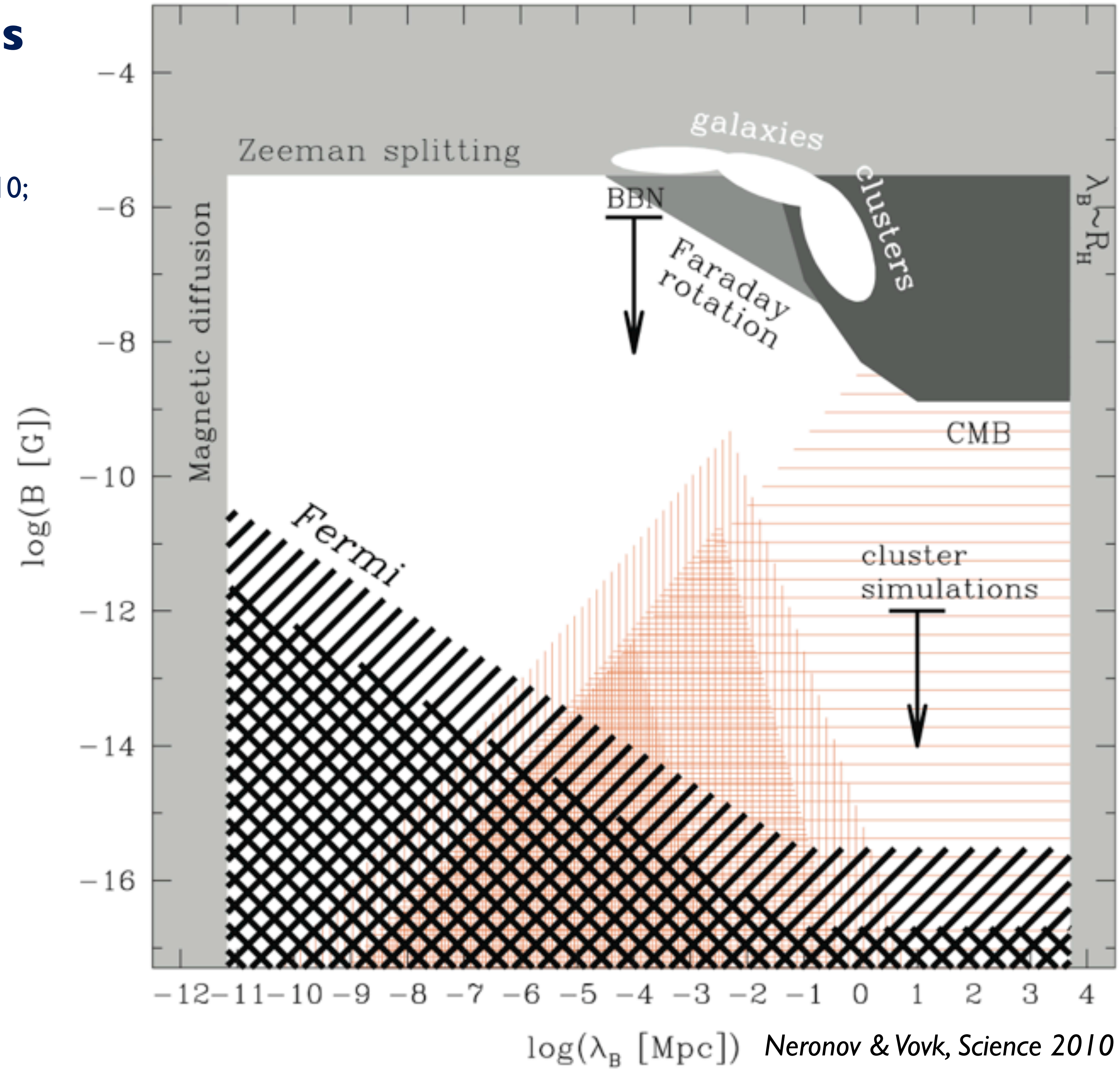


magnetic polarization measurements in the Pipe nebula
F.O.Alves, Franco, Girart 2008

Observed Magnetic Fields

Cosmic Magnetic Fields

- **Lower** limits ~ 10^{-15} G
(FERMI, HESS Obs. e.g. Neronov & Vovk 2010; Taveccio et al. 2010)
- Upper limits:
  - BBN ~ 10^{-7} G (Grasso & Rubinstein 2001)
  - CMBR ~ 10^{-9} G
  - Reionization ~ 10^{-9} G
(Schleicher et al. 2008)
- Cluster and galactic fields ~ μG (e.g. Beck 1999)

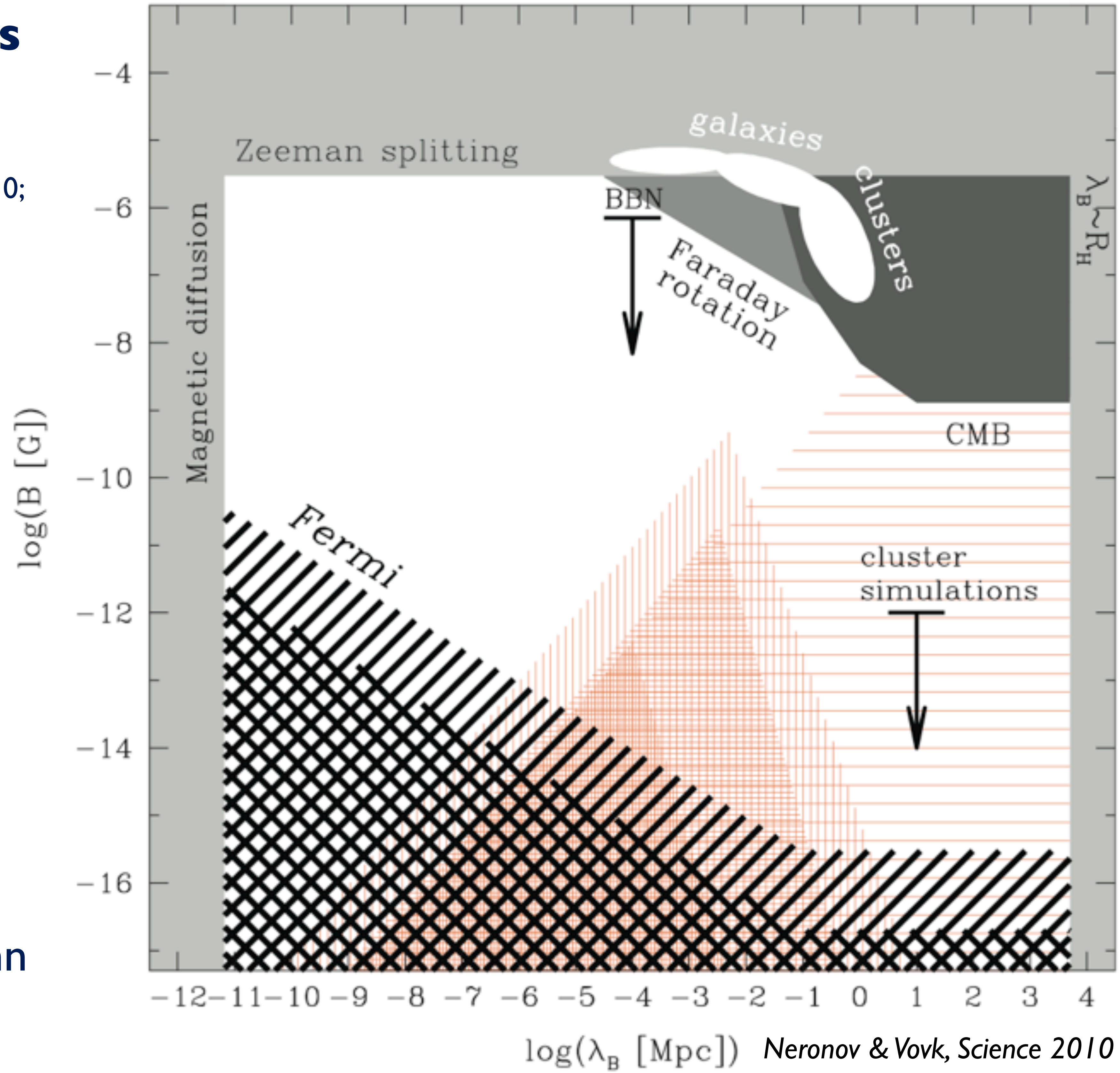


Neronov & Vovk, Science 2010

Observed Magnetic Fields

Cosmic Magnetic Fields

- **Lower** limits ~ 10^{-15} G
(FERMI, HESS Obs. e.g. Neronov & Vovk 2010; Taveccio et al. 2010)
- Upper limits:
  - BBN ~ 10^{-7} G (Grasso & Rubinstein 2001)
  - CMBR ~ 10^{-9} G
  - Reionization ~ 10^{-9} G (Schleicher et al. 2008)
- Cluster and galactic fields ~ μG (e.g. Beck 1999)



Neronov & Vovk, Science 2010

Origin?

- primordial (e.g. inflation, cosmic PT)
- astrophysical (e.g. Biermann battery, Weibel instability)

Generation of Primordial Fields

Possible generation of primordial magnetic fields
(e.g. *Grasso & Rubinstein 2001*; *Widrow 2003*; *Widrow et al. 2011*)

- during cosmic inflation (e.g. *Turner & Widrow 1998*)

- during cosmic phase transitions
  - electroweak PT ($t \sim 10^{-10}$ sec, $T \sim 100$ GeV)
    (e.g. *Baym et al. 1996*)
  - QCD PT ($T \sim 100$ MeV)
  (e.g. *Quashnock et al. 1989; Cheng & Olinto 1994; Sigl et al. 1997*)

Generation of Primordial Fields

Possible generation of primordial magnetic fields
(e.g. *Grasso & Rubinstein 2001*; *Widrow 2003*; *Widrow et al. 2011*)

- during cosmic inflation (e.g. *Turner & Widrow 1998*)

- during cosmic phase transitions
  - electroweak PT ($t \sim 10^{-10}$ sec, $T \sim 100$ GeV)
    (e.g. *Baym et al. 1996*)
  - QCD PT ($T \sim 100$ MeV)
    (e.g. *Quashnock et al. 1989; Cheng & Olinto 1994; Sigl et al. 1997*)

⇒ causal process
⇒ coherence length limited by Hubble length
at epoch of generation

Evolution of Primordial Fields

Subsequent evolution

- dilution by cosmic expansion:

$$B \propto a^{-2}$$

assumption: flux freezing (no dynamic damping/ amplification)

- but: damping/amplification is important
(Jedamzik et al. 98, Subramanian & Barrow 98, Banerjee & Jedamzik 2003/2004, Schleicher et al. 2010, Sur et al. 2010)

Evolution of Primordial Fields

MHD equations on an expanding background:

$$\frac{\partial \rho}{\partial t} + \frac{1}{a} \nabla \cdot \Big((\rho + p)\, \mathbf{v} \Big) + 3\, H\, (\rho + p) = \frac{6}{a} H\, \xi\, (\nabla \cdot \mathbf{v}) - \chi\, \nabla \cdot \mathbf{q}\,,$$

$$\frac{1}{a} \left(\frac{\partial}{\partial t} + \frac{1}{a} (\mathbf{v} \cdot \nabla) + H \right) \mathbf{v} + \frac{1}{a} \frac{\mathbf{v}}{\rho + p} \frac{\partial p}{\partial t} + \frac{1}{a^2} \frac{\nabla p}{\rho + p} + \frac{1}{a^2} \left(\frac{\mathbf{B} \times (\nabla \times \mathbf{B})}{4\pi\, (\rho + p)} \right) =$$
$$\frac{1}{a^3} \frac{\nu}{\rho + p} \left(\nabla^2 \mathbf{v} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{v}) \right) + \frac{1}{a^3} \frac{\xi}{\rho + p} \nabla (\nabla \cdot \mathbf{v}) - \frac{1}{\rho + p} \left(\frac{\partial}{\partial t} + 5\, H \right) \chi\, \mathbf{q}\,,$$

$$\frac{1}{a} \left(\frac{\partial}{\partial t} + 2\, H \right) \mathbf{B} = \frac{1}{a^2} \nabla \times (\mathbf{v} \times \mathbf{B})$$

- ν, ξ viscosity; χ, $\mathbf{q}$ heat conductivity/heat flux
- H, a Hubble paramter/scale factor $\Longrightarrow$ cosmic expansion

assumption: infinite conductivity $\Longrightarrow$ large Prandtl numbers

Evolution of Primordial Fields

MHD equations on an expanding background with *super co-moving* variables* (e.g. *Enqvist 98*):

$$
\begin{aligned}
\frac{\partial \tilde\rho}{\partial \tilde t} + \nabla\cdot(\tilde\rho\,\tilde{\mathbf{v}}) &= 0 \quad ,\\
\frac{\partial \tilde{\mathbf{v}}}{\partial \tilde t} + (\tilde{\mathbf{v}}\cdot\nabla)\,\tilde{\mathbf{v}} + \frac{1}{\tilde\rho}\nabla\tilde p + \frac{1}{4\pi\tilde\rho}\tilde{\mathbf{B}}\times(\nabla\times\tilde{\mathbf{B}}) &= -\tilde{\mathbf{s}} \quad ,\\
\frac{\partial \tilde\epsilon}{\partial \tilde t} + \nabla\cdot(\tilde\epsilon\,\tilde{\mathbf{v}}) + \tilde p\,(\nabla\cdot\tilde{\mathbf{v}}) &= -\tilde\Gamma \quad ,\\
\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde t} - \nabla\times(\tilde{\mathbf{v}}\times\tilde{\mathbf{B}}) &= 0 \quad .
\end{aligned}
$$

$\Longrightarrow$ no Hubble-expansion

$\Longrightarrow$ same form than non-relativistic MHD equations

*
$$
\begin{aligned}
\tilde\rho &\equiv \rho\,a^3 & \tilde p &\equiv p\,a^4 & \tilde{\mathbf{B}} &\equiv \mathbf{B}\,a^2\\
\tilde{\mathbf{v}} &\equiv \mathbf{v}\,a^{1/2} & \tilde\epsilon &\equiv \epsilon\,a^4 & \tilde T &\equiv T\,a^2\\
\tilde\chi &\equiv \chi\,a^{3/2} & \tilde\nu &\equiv \nu\,a^{5/2} & d\tilde t &\equiv dt\,a^{-3/2}\\
\tilde\xi &\equiv \xi\,a^{5/2} & \tilde H &\equiv a^{3/2}\,H & &
\end{aligned}
$$

Evolution of Primordial Fields

Further assumptions:

- incompressible MHD:

  $v,\ v_A \ll v_s$

  $v_A < v_s$ if $B < 5\mathrm{x}10^{-5}$ G at recombination

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} - (\mathbf{v}_A\cdot\nabla)\mathbf{v}_A = \mathbf{f},$$

$$\frac{\partial \mathbf{v}_A}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}_A - (\mathbf{v}_A\cdot\nabla)\mathbf{v} = \nu\nabla^2\mathbf{v}_A,$$

dissipation:

$$\mathbf{f} = \begin{cases} \eta\nabla^2\mathbf{v} & l_{\mathrm{mfp}} \ll l, \\ -\alpha\mathbf{v} & l_{\mathrm{mfp}} \gg l, \end{cases}$$

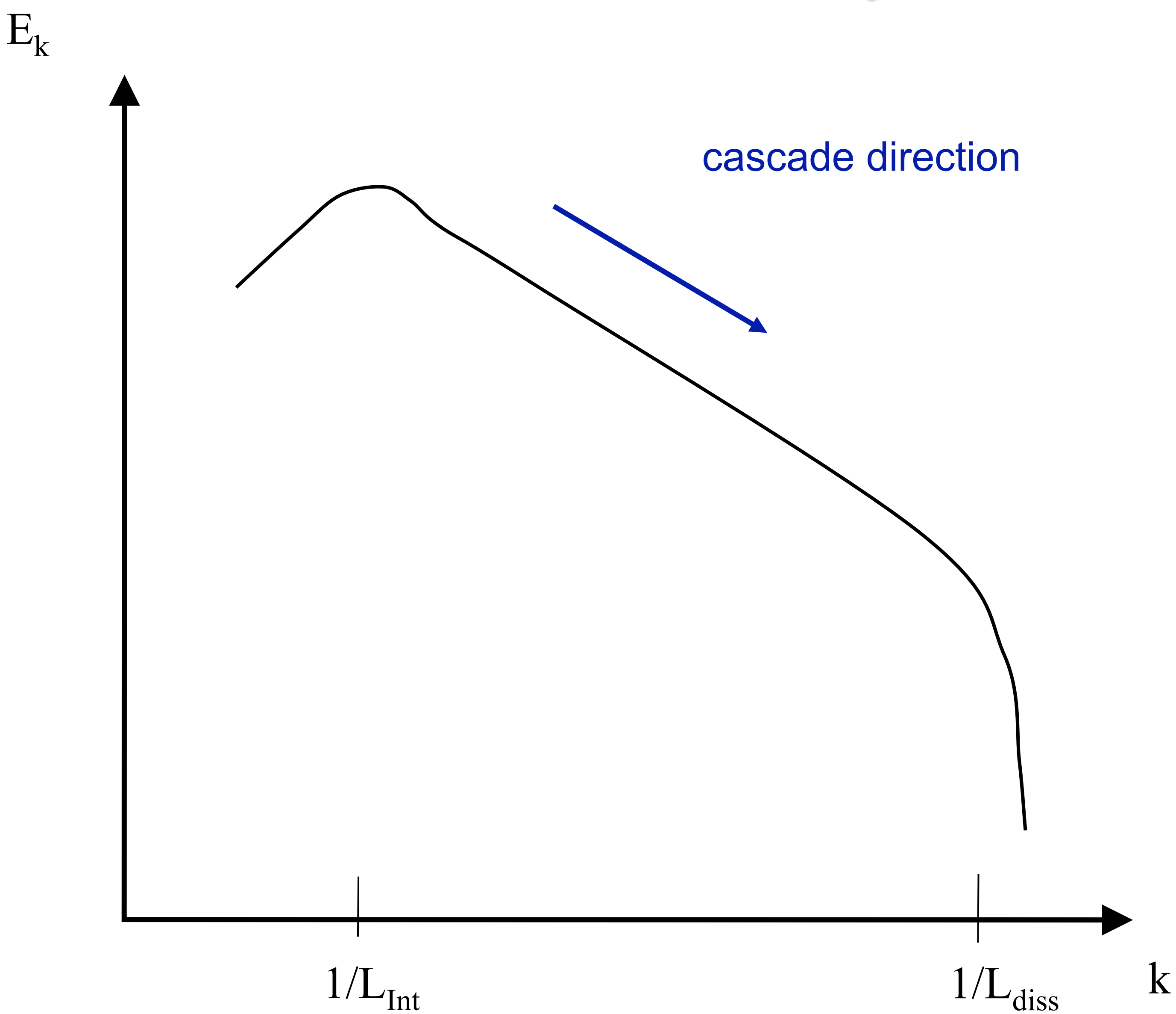
Reynolds number:

$$R_e(l) = \frac{v^2/l}{|\mathbf{f}|} = \begin{cases} \frac{vl}{\eta} & l_{\mathrm{mfp}} \ll l \\ \frac{v}{\alpha l} & l_{\mathrm{mfp}} \gg l, \end{cases}$$

Evolution of Primordial Fields

$\Longrightarrow$ decay via MHD turbulence

$$E \approx \int d\ln k k^3(\langle|v_k|^2\rangle + \langle|v_{A,k}|^2\rangle) \equiv \int d\ln k E_l,$$



quasi-stationary transfer of energy in *k*-space

$\Longrightarrow$ Kolmogorov Turbulence

$$\frac{dE_l}{dt} \approx \frac{E_l}{\tau_l} \approx \text{const}(l)$$

Evolution of Primordial Fields

spectra of turbulence

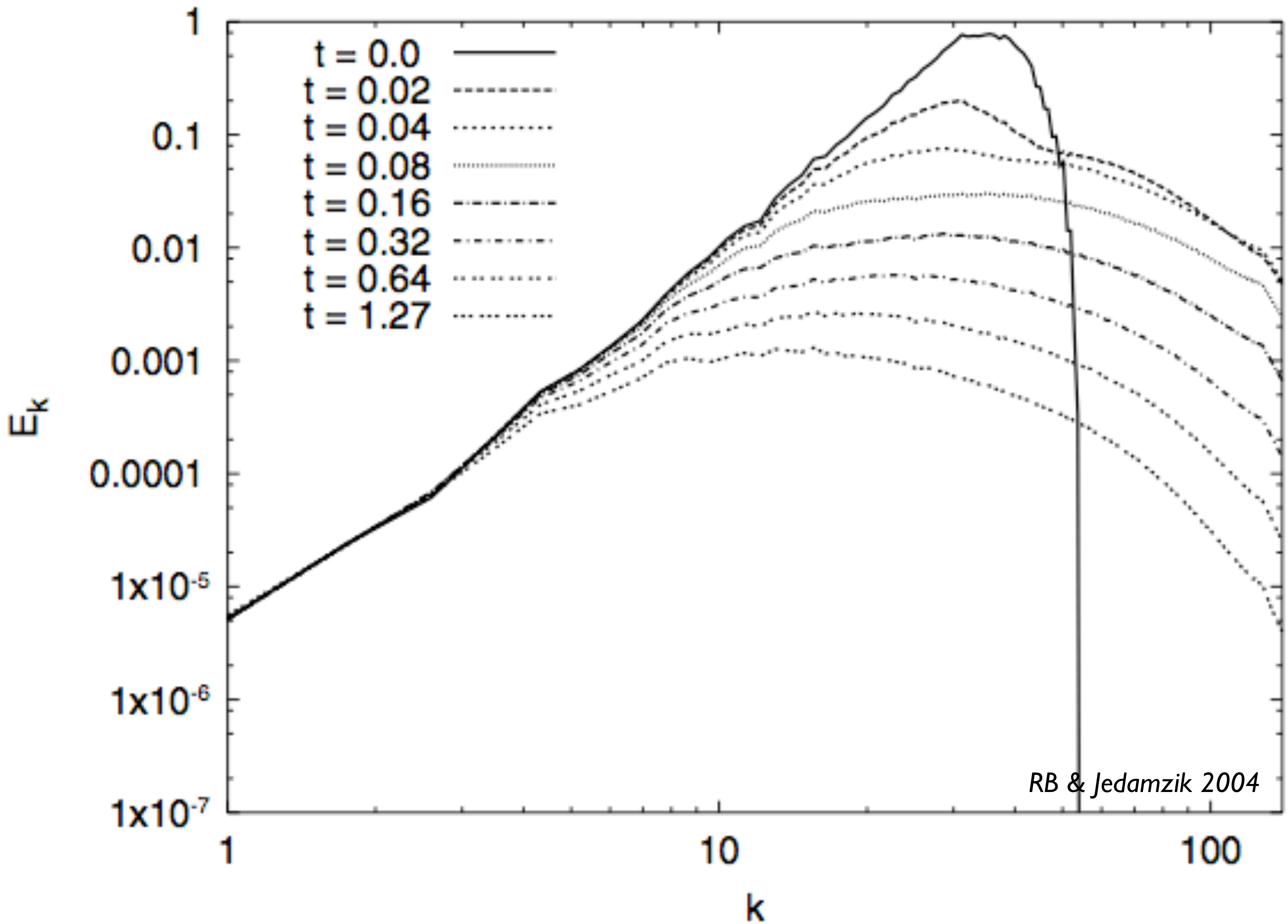
Kolmogorov ('41): $\tau_{\rm K} = l/v_l$

Iroshnikov-Kraichnan ('64/'65): $\tau_{\rm IK} = (l/v_l)\,(v_{{\rm A},L}/v_l)$

$$E_k/k \propto \begin{cases} k^{-5/3} & : \quad \text{unmagnetized} \\ k^{-3/2} & : \quad \text{magnetized} \end{cases}$$

Evolution of primordial fields

turbulent decay: numerical simulations



Evolution of Primordial Fields

decay laws

- assume initial spectrum on large scales ($l > \mathrm{L}$):

$$E_k \approx E_0\left(\frac{k}{k_0}\right)^n = E_0\left(\frac{l}{L_0}\right)^{-n} \quad \text{for } l > L_0.$$

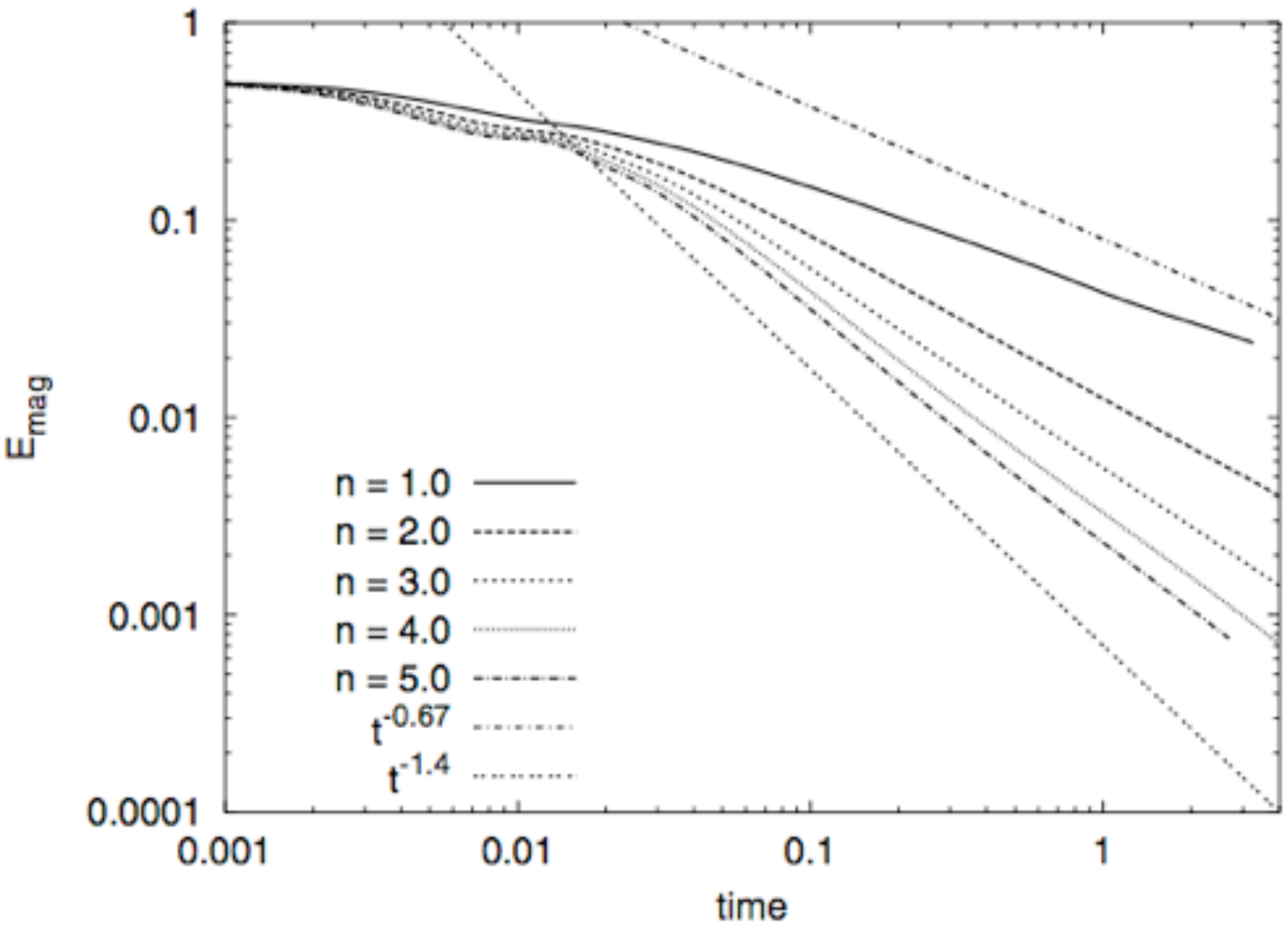
- with: $v_l = \sqrt{E_l}$:

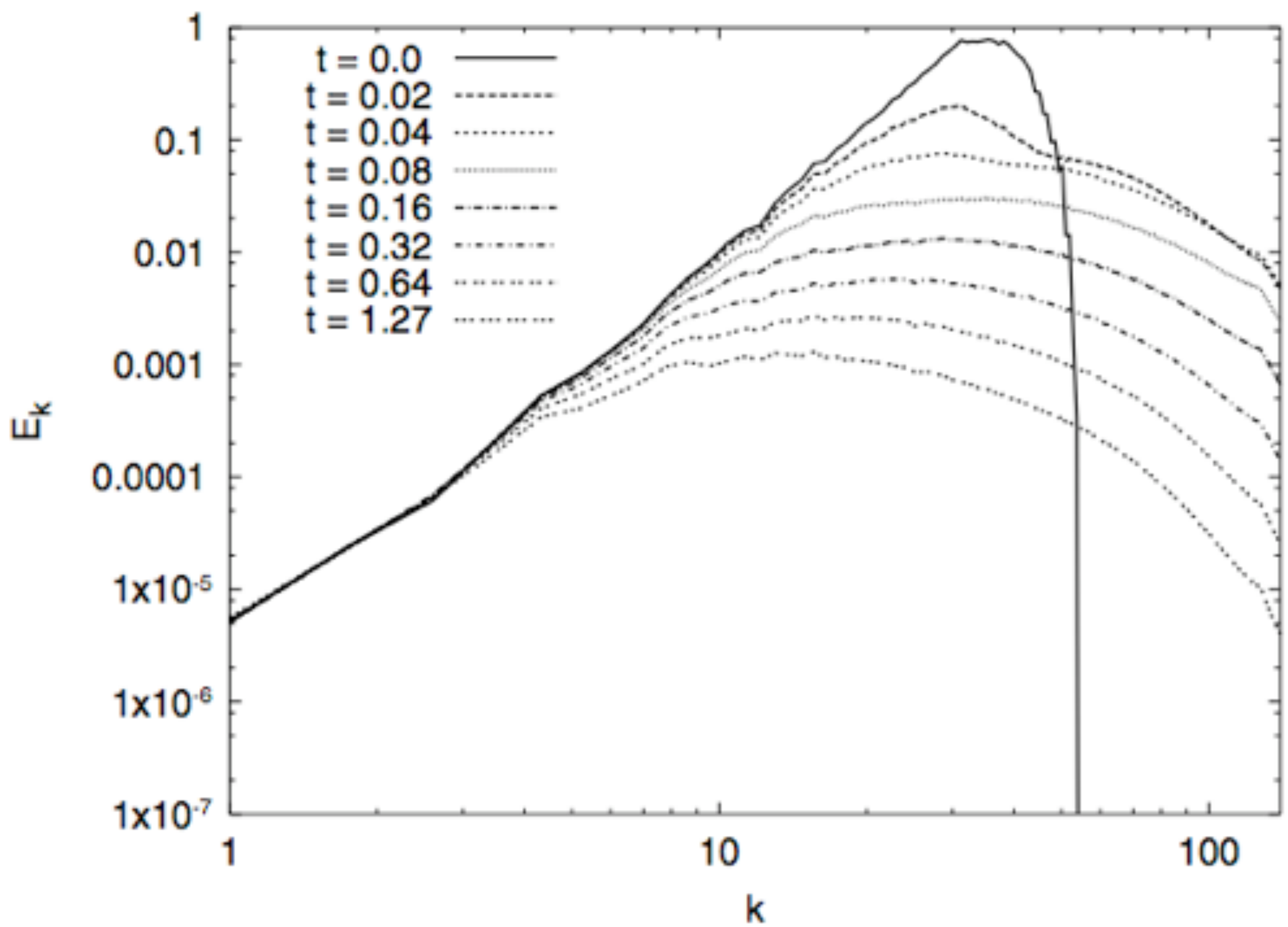
energy decay: $$E \approx E_0\left(\frac{t}{\tau_0}\right)^{-2n/(2+n)}$$

increase of coherence length: $$L \approx L_0\left(\frac{t}{\tau_0}\right)^{2/(2+n)}$$

Evolution of primordial fields

decay laws





- decay law: $$E \approx E_0 \left(\frac{t}{\tau_0} \right)^{-2n/(2+n)}$$

- growths of coherence length: $$L \approx L_0 \left(\frac{t}{\tau_0} \right)^{2/(2+n)}$$

Evolution of Primordial Fields

Helical Fields

- Helicity (measures complexity of the field):

$$\mathcal{H} \equiv \frac{1}{V} \int_V \mathrm{d}^3 x \, \mathbf{A} \cdot \mathbf{B}$$

is conserved (no resistivity)

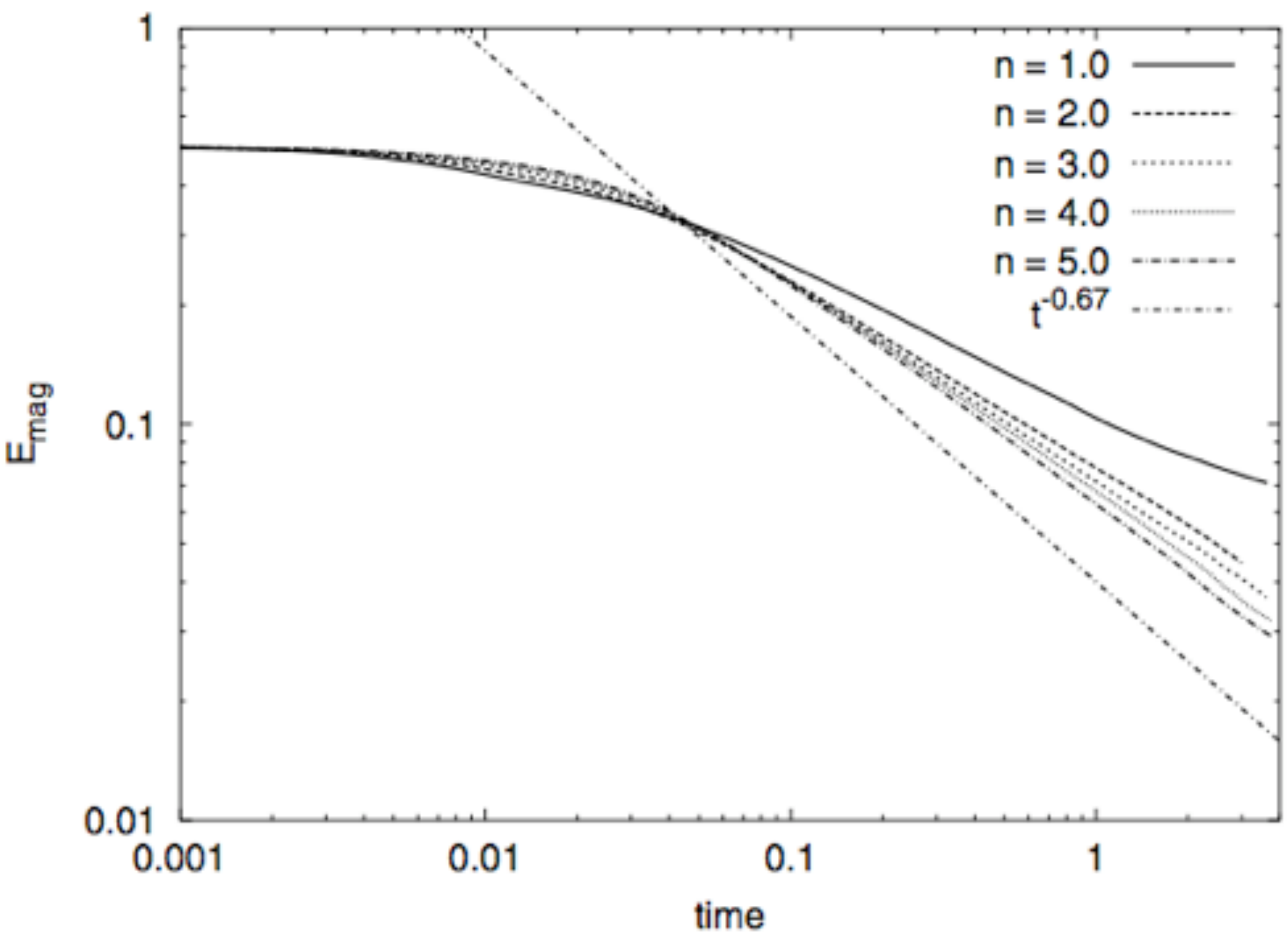
- maximal helical field: $H \sim B^{\wedge 2} \, L \approx E \, L$

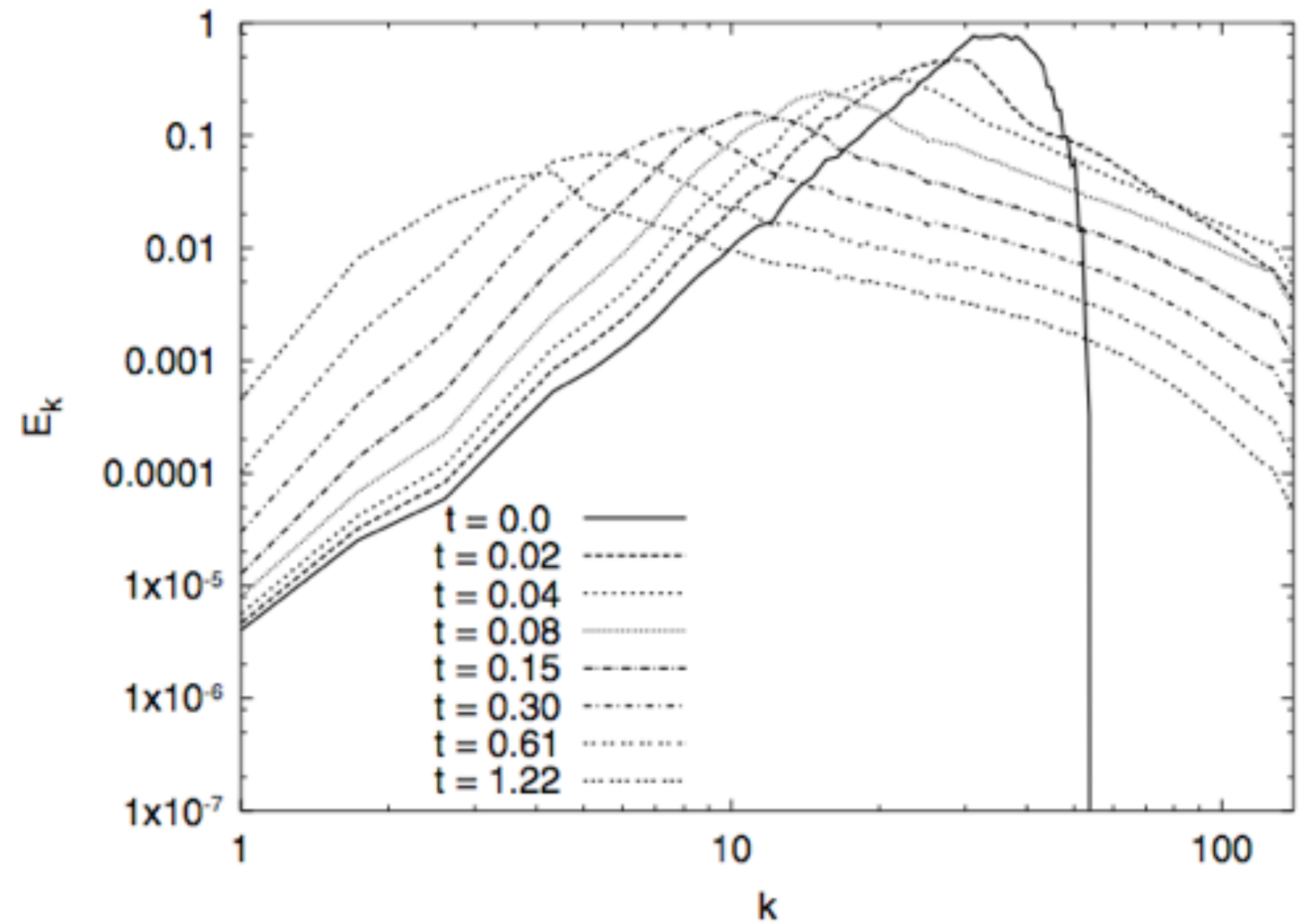
energy decay: $$E \approx E_0 \left(\frac{t}{\tau_0} \right)^{-2/3}$$

inverse cascade: $$L \approx L_0 \left(\frac{t}{\tau_0} \right)^{2/3}$$

Evolution of primordial fields

- Fields with maximum helicity: $\mathcal{H}_{\max} \approx \langle B^2 L \rangle \approx (8\pi) E L$



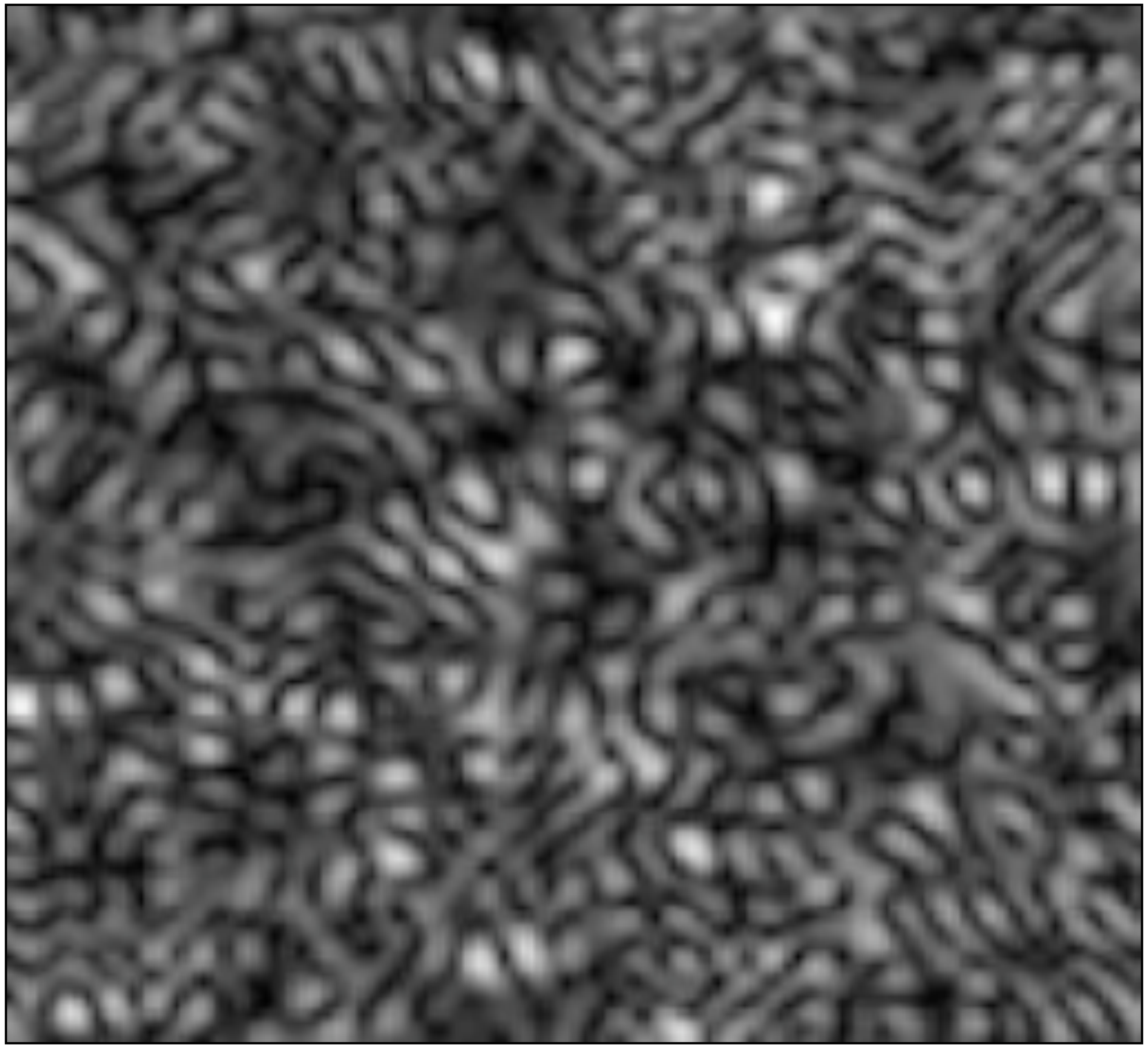


- decay law: $E \approx E_0 \left(\frac{t}{\tau_0} \right)^{-2/3}$

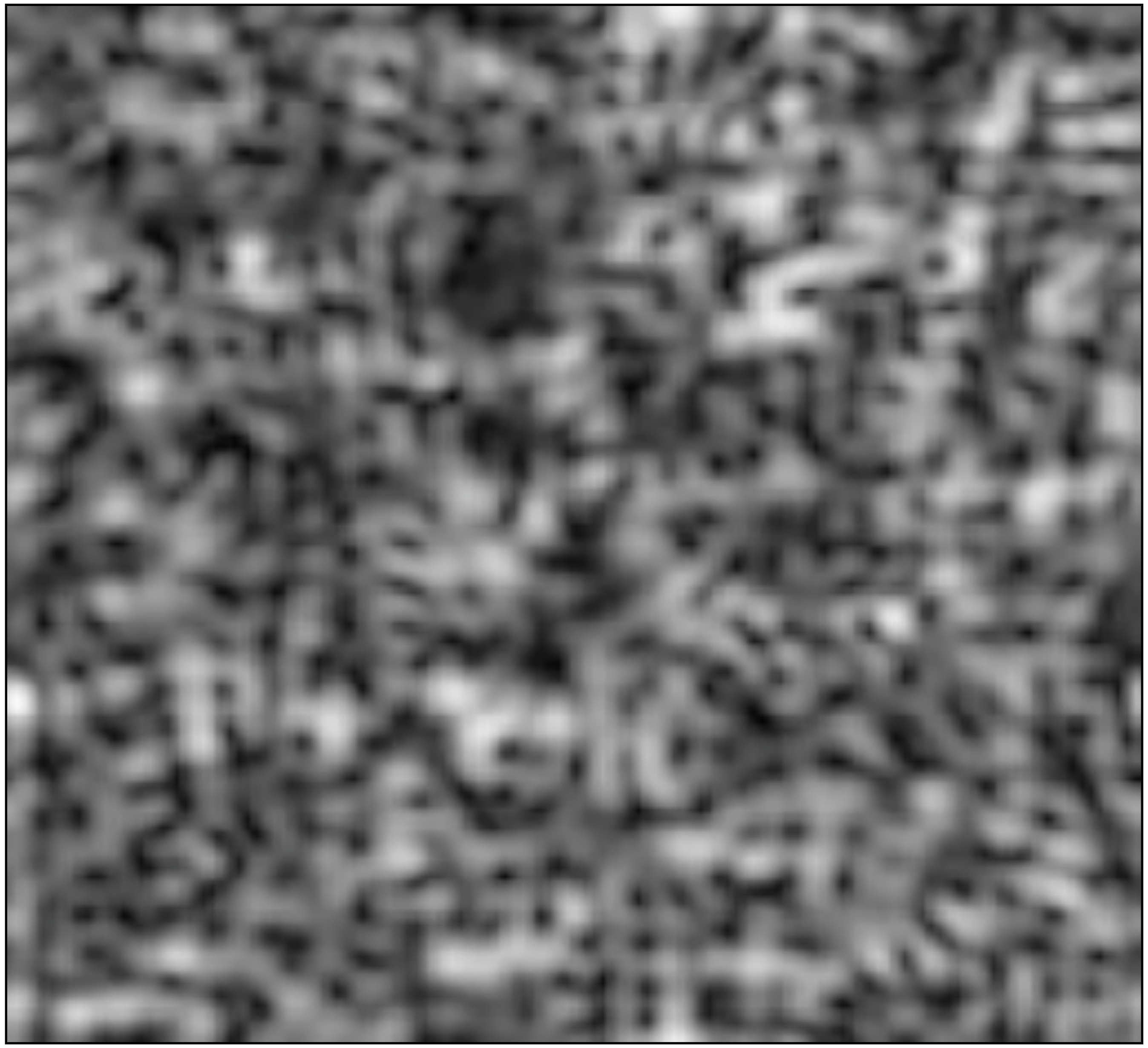
- growths of coherence length: $L \approx L_0 \left(\frac{t}{\tau_0} \right)^{2/3}$ inverse cascade

Evolution of primordial fields

Evolution of small scale random magnetic fields



no initial helicity



with max. initial helicity

Evolution of primordial fields

Evolution equation:

$$\tau_L \approx \frac{L(T)}{v_L(T)} \approx \frac{1}{H(T)} \approx t_{\mathrm{H}}$$

turbulent regime ($\mathrm{R_e} \gg 1$): $$v_L(T) = v_{\mathrm{A},L}(T)$$

viscous regime ($\mathrm{R_e} < 1$):

$$v_L(T) = \frac{v_{\mathrm{A},L}^2(T)\,L}{\eta(T)}$$

for $l_{\mathrm{mfp}} \ll L$

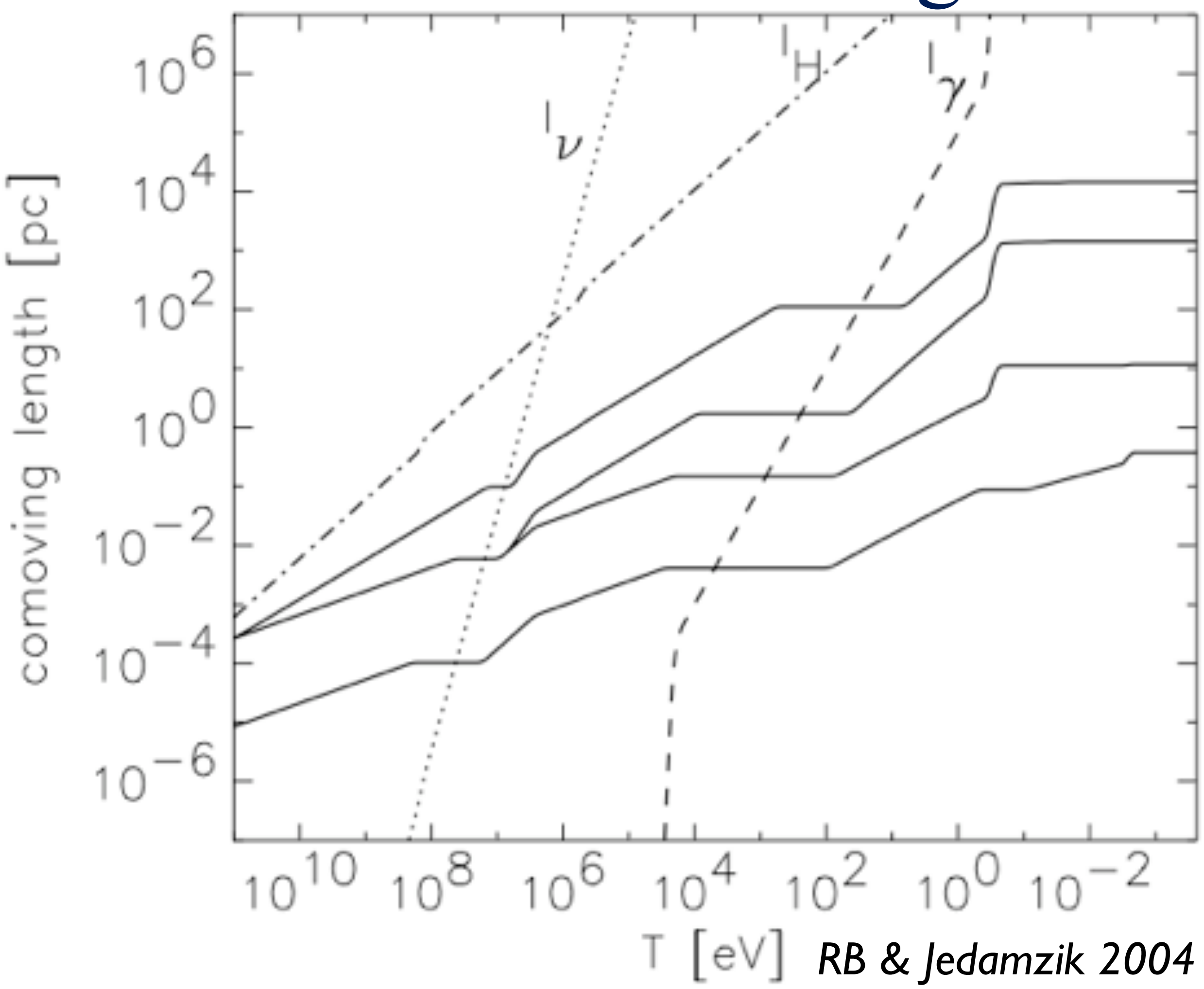
$$v_L(T) = \frac{v_{\mathrm{A},L}^2(T)}{\alpha(T)\,L}$$

for $l_{\mathrm{mfp}} \gg L$

Evolution of primordial fields

combine with cosmic evolution

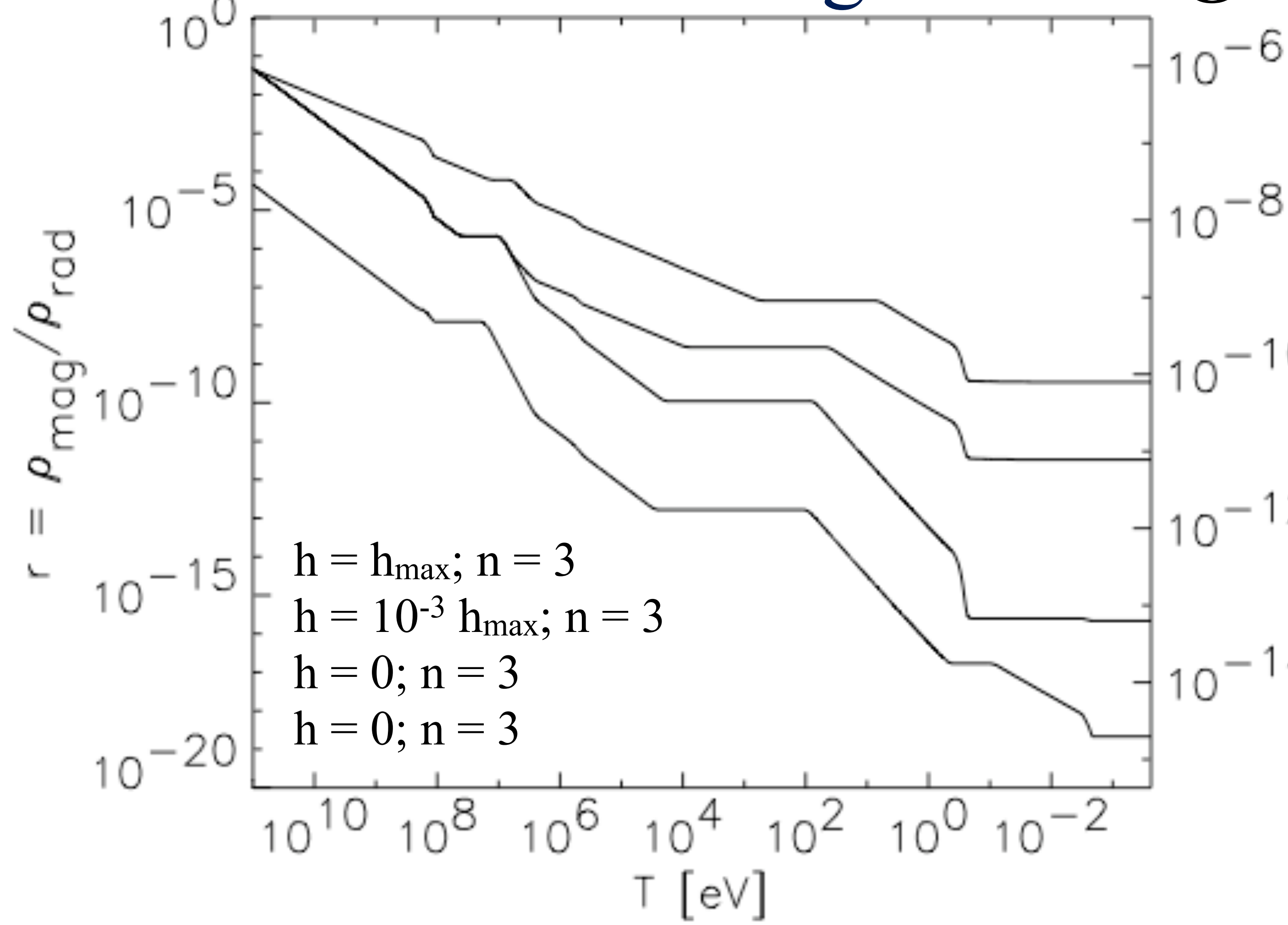
coherence length



RB & Jedamzik 2004

field strength

G

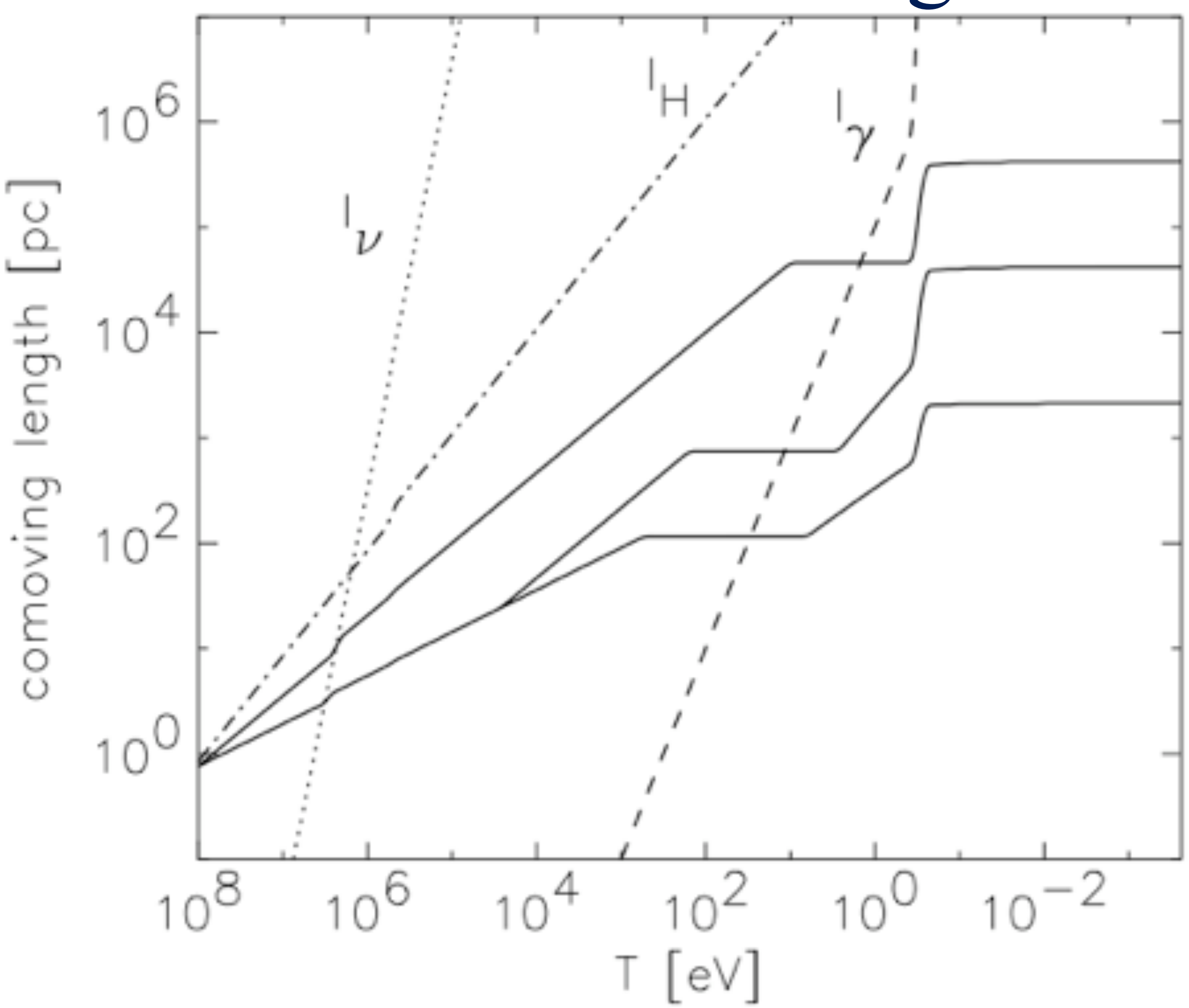


assume magneto-genesis at EW-PT (T_{gen} = 100 GeV)

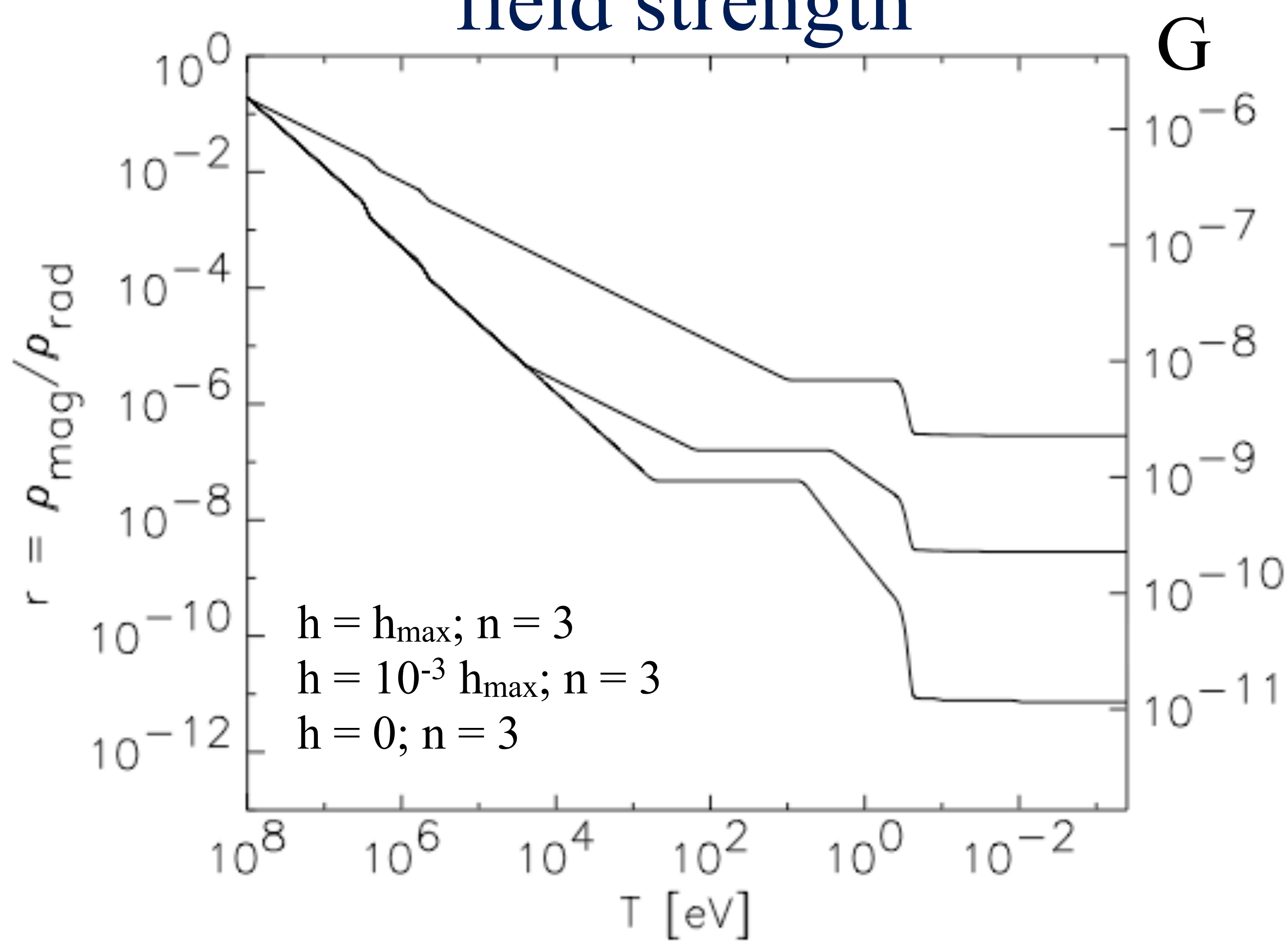
Evolution of primordial fields

combine with cosmic evolution

coherence length



field strength

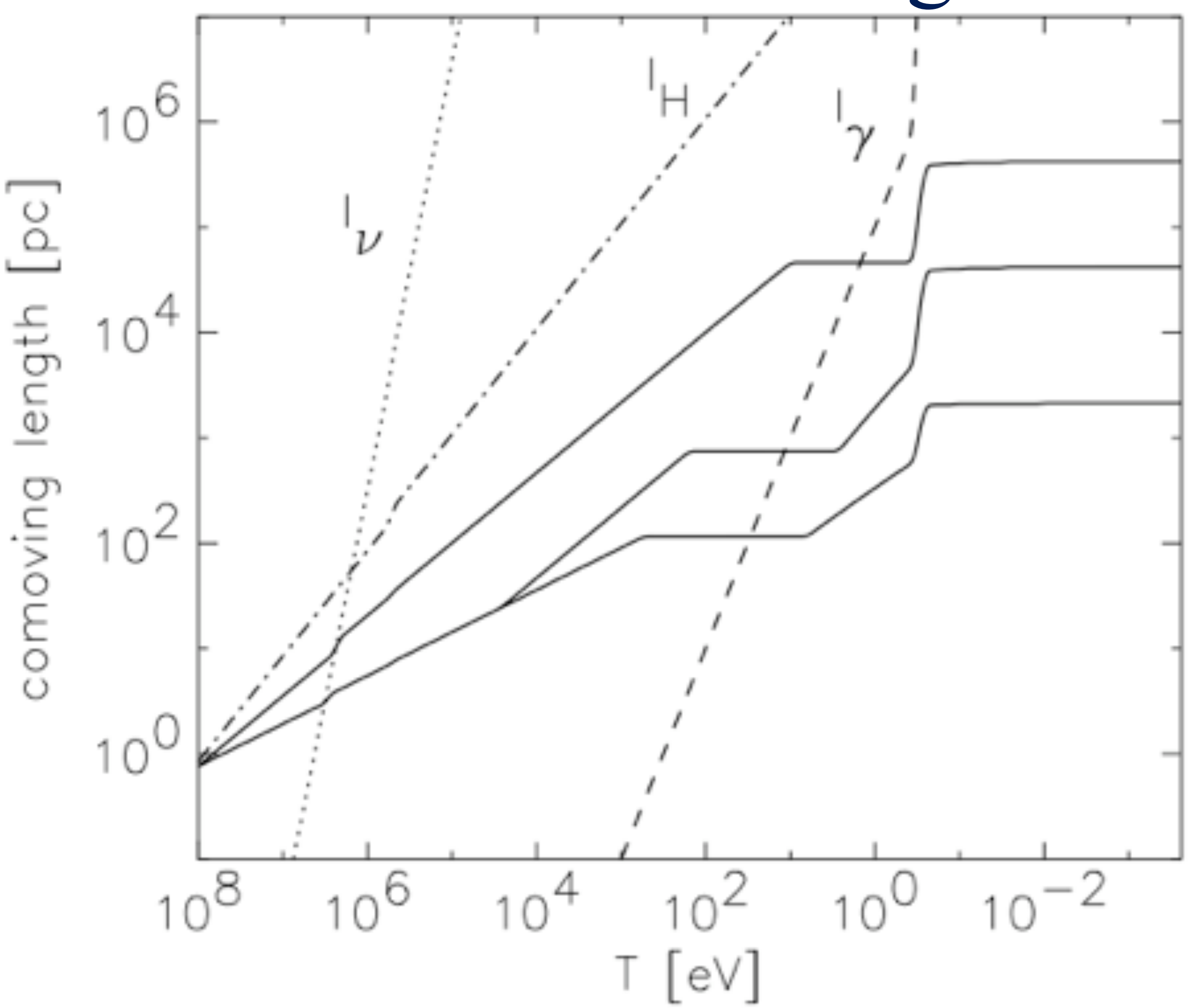


assume magneto-genesis at QCD-PT (T_{gen} = 100 MeV)

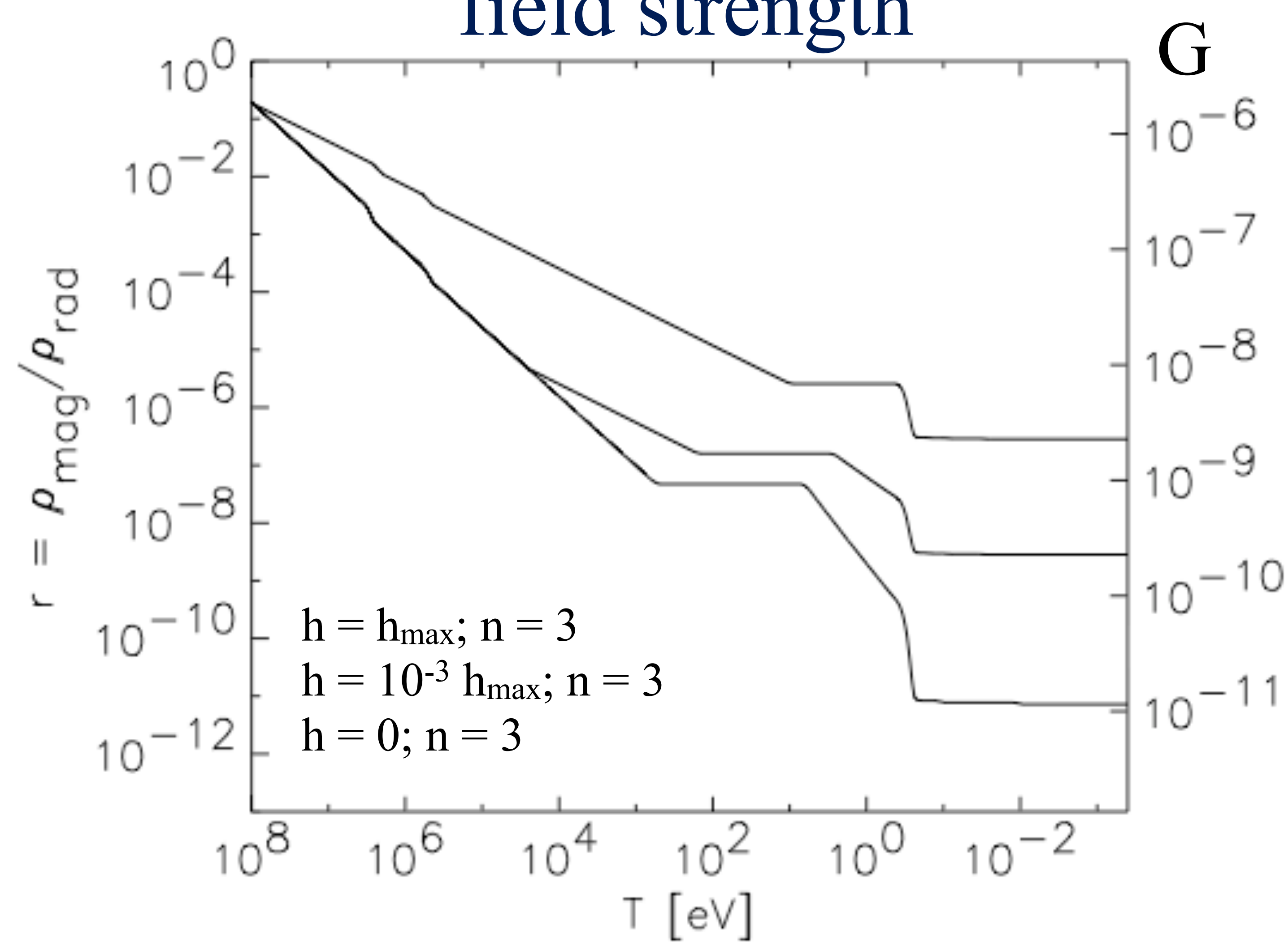
Evolution of primordial fields

combine with cosmic evolution

coherence length



field strength

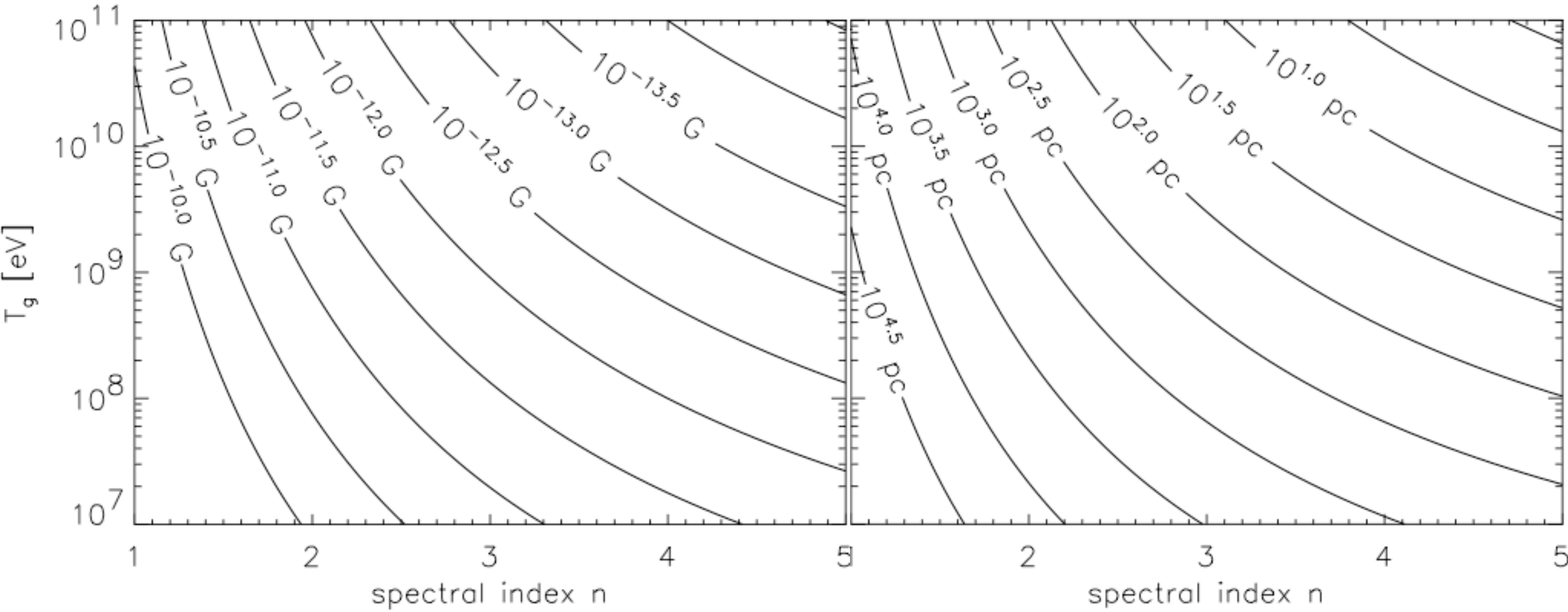


assume magneto-genesis at QCD-PT (T_{gen} = 100 MeV)

Cluster fields of primordial origin?

Evolution of primordial fields

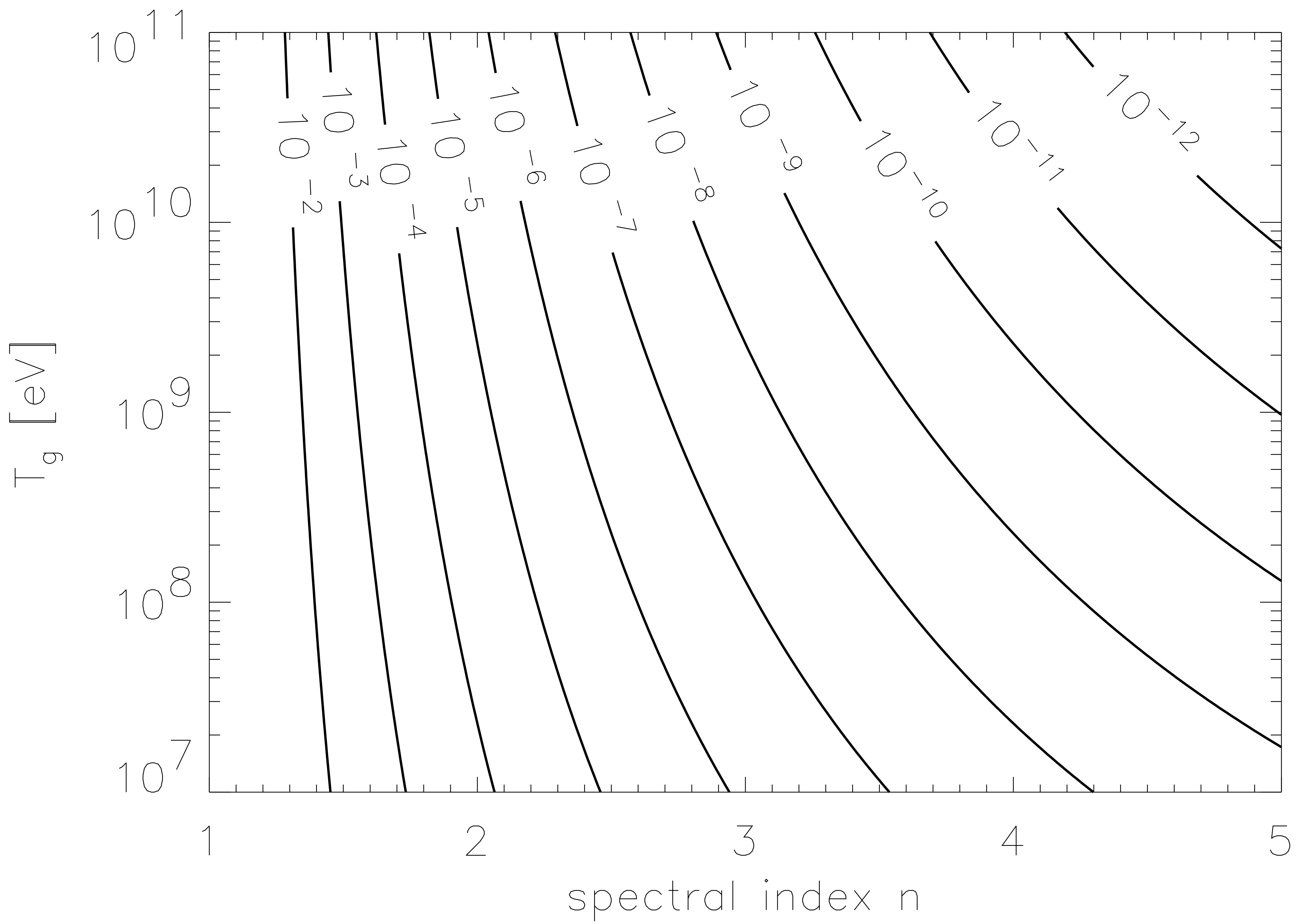
combine with cosmic evolution



present day field strength and coherence length

Evolution of primordial fields

combine with cosmic evolution



minimum initial helicity needed for a fully helical field today

Effects of primordial fields

- Magnetic Jeans mass:

  (Subramanian & Barrow 1998)

$$M_J^B \sim 10^{10} M_\odot \left(\frac{B_0}{3\ \mathrm{nG}}\right)^3$$

- Ambipolar diffusion heating:

$$L_{\mathrm{AD}} = \frac{\eta_{\mathrm{AD}}}{4\pi} \left| \left(\nabla \times \vec{B}\right) \times \vec{B}/B \right|^2$$

  (Sethi & Subramanian 2005,
  Schleicher, Banerjee & Klessen 2008)

- Smallest scale:

  (Jedamzik et al. 1998,
  Subramanian & Barrow 1998)

$$k_{\max} \sim 234\ \mathrm{Mpc}^{-1} \left(\frac{B_0}{1\ \mathrm{nG}}\right)^{-1} \left(\frac{\Omega_m}{0.3}\right)^{1/4} \times \left(\frac{\Omega_b h^2}{0.02}\right)^{1/2} \left(\frac{h}{0.7}\right)^{1/4},$$

Effects of primordial fields

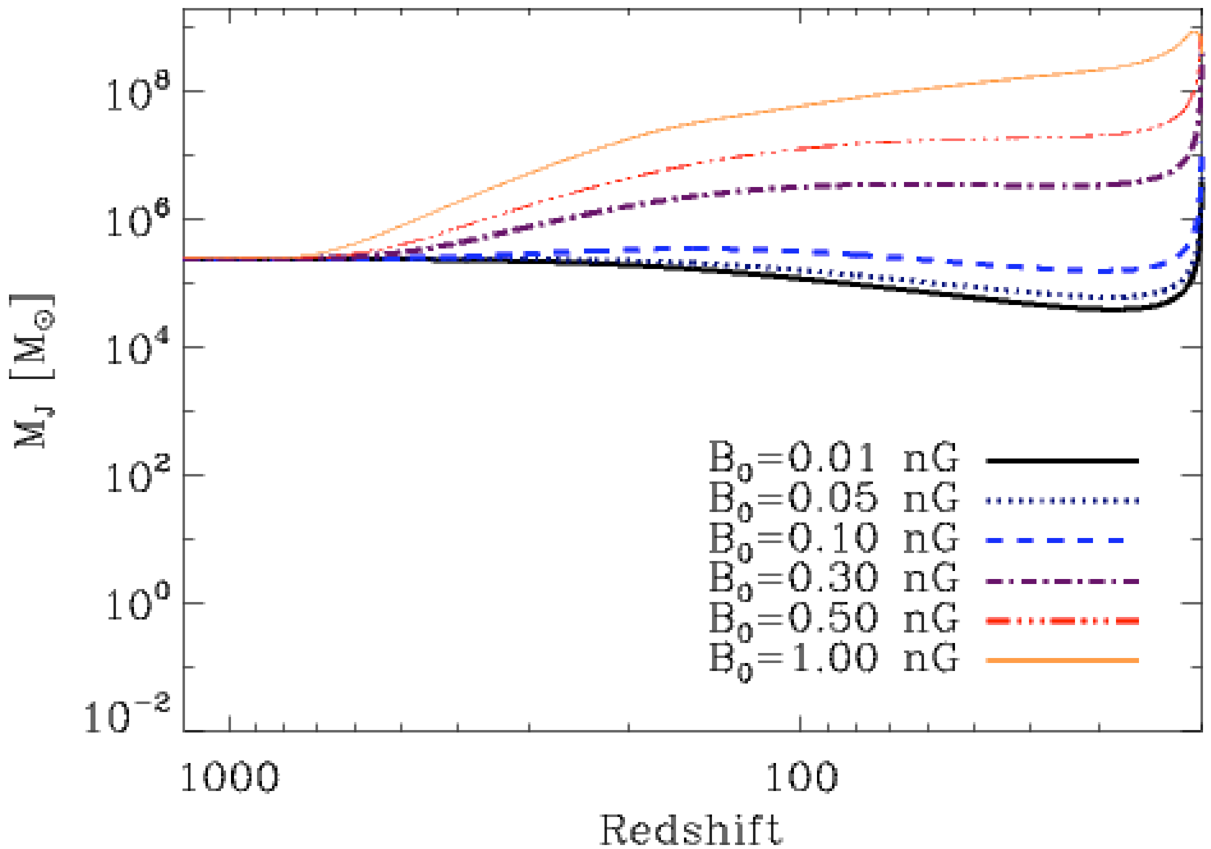
Ambipolar Diffusion

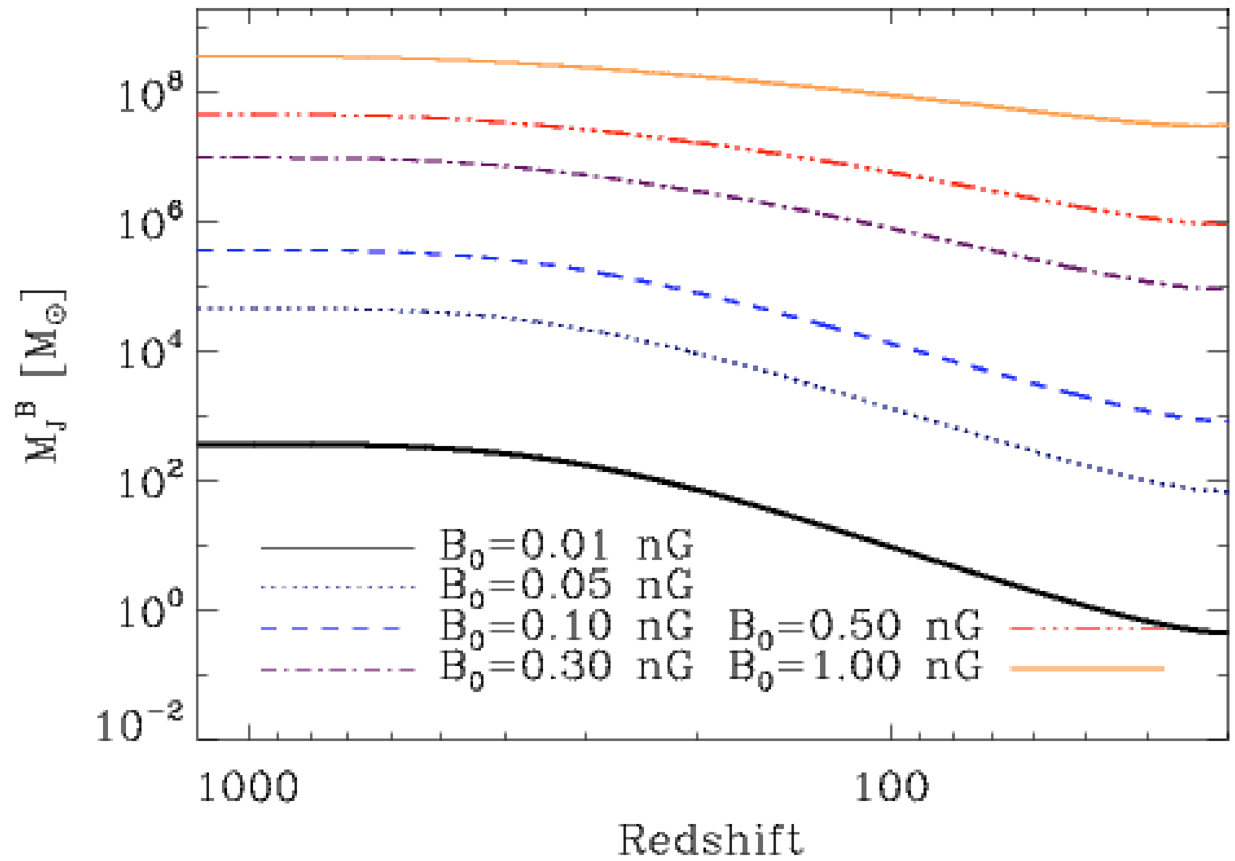
- Ions are coupled to the magnetic field.
- Neutrals are indirectly coupled to the magnetic field by collisions with the ions.
- The coupling is not perfect: Sometimes they diffuse through the field lines
- Magnetic energy can be dissipated by friction between ions and neutrals.

$$L_{\mathrm{ambi}} = \frac{\rho_n}{16\pi^2 \gamma \rho_b^2 \rho_i} \left| \left(\nabla \times \vec{B} \right) \times \vec{B} \right|^2$$

$$\gamma = \frac{\frac{1}{2} n_H \langle \sigma v \rangle_{\mathrm{H^+,H}} + \frac{4}{5} n_{\mathrm{He}} \langle \sigma v \rangle_{\mathrm{H^+,He}}}{m_H \left[n_{\mathrm{H}} + 4 n_{\mathrm{He}} \right]}$$

Effects of primordial fields



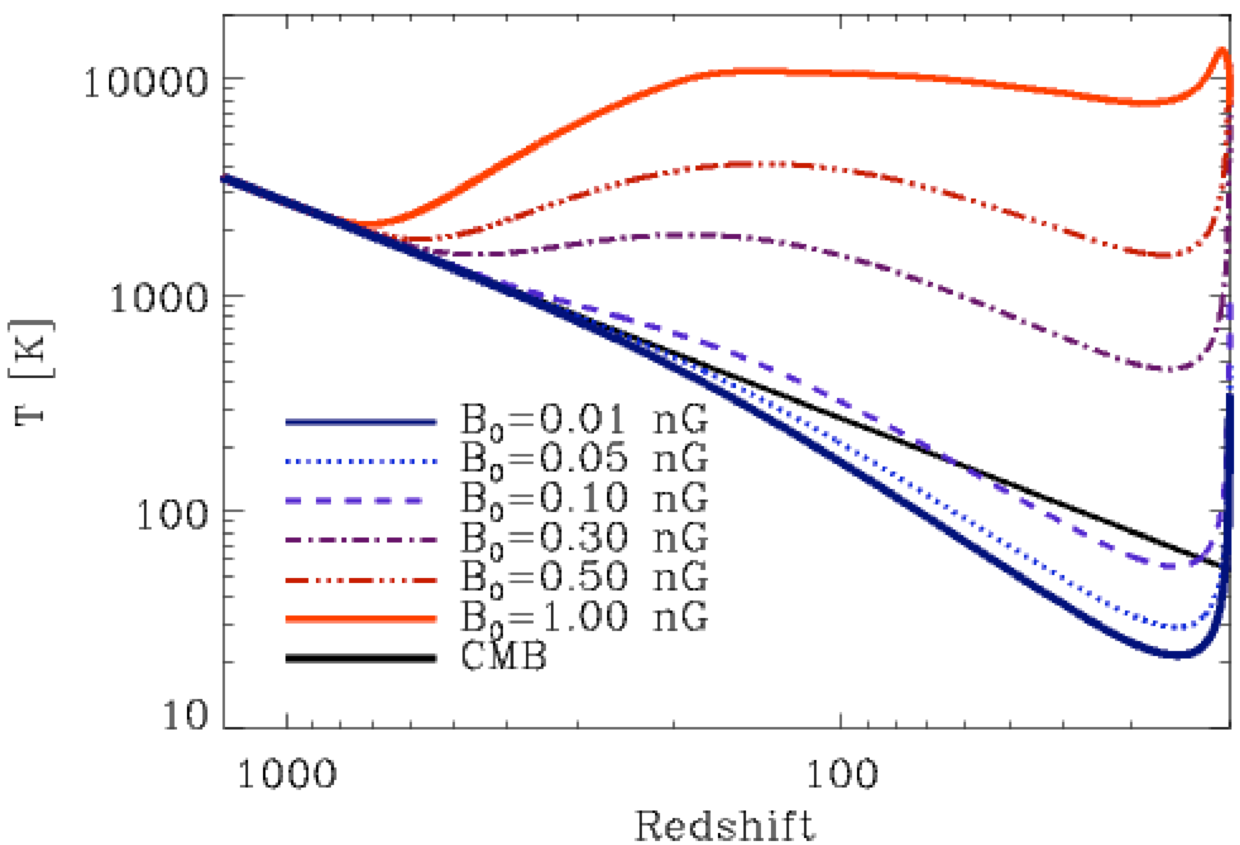


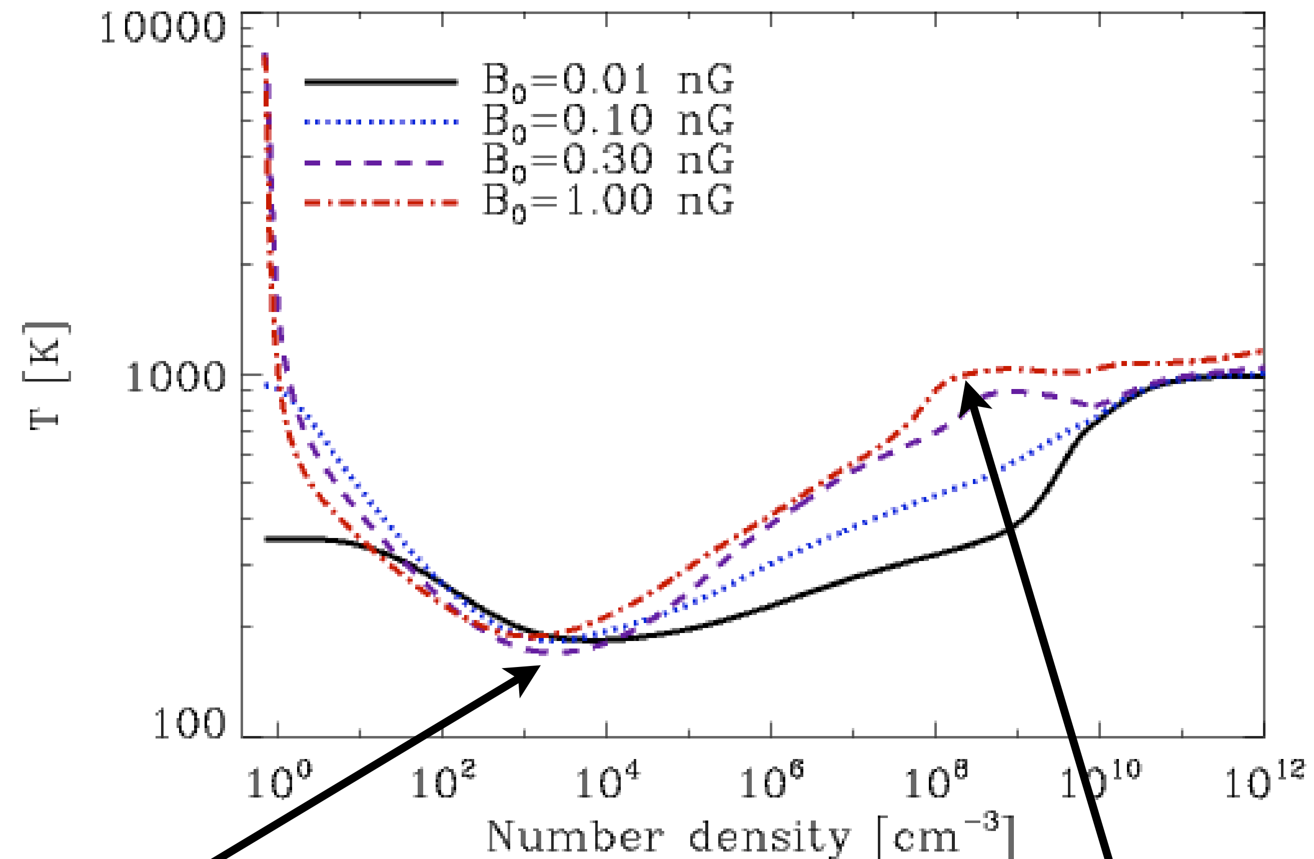
Thermal / magnetic Jeans masses: Critical mass scale for gravity to overcome thermal / magnetic pressure.
Both are significantly increased in the presence of strong magnetic fields.

Schleicher et al. (2009)

Effects of primordial fields

Ambipolar diffusion:





Primordial magnetic fields increase gas temperature

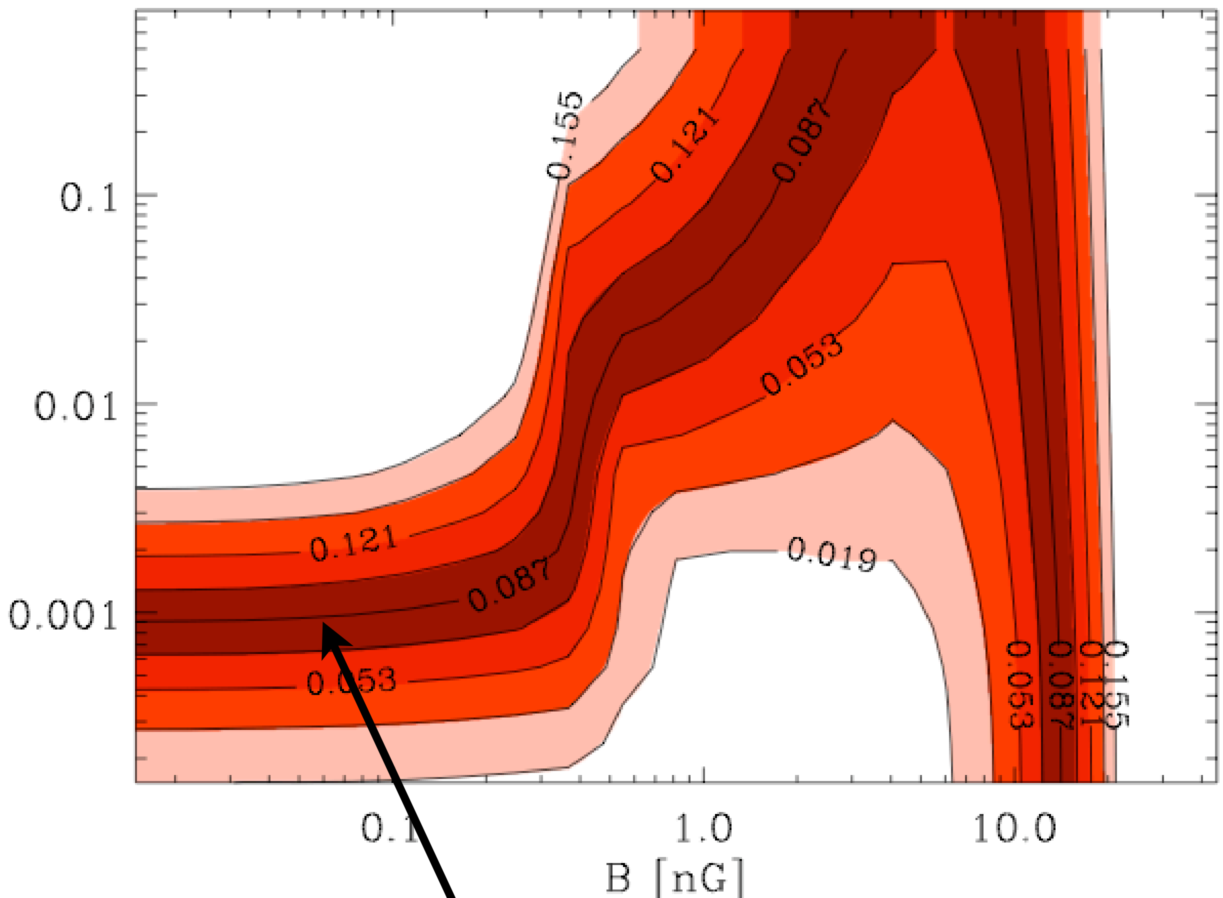
same fragmentation scale

higher temperature -> more accretion

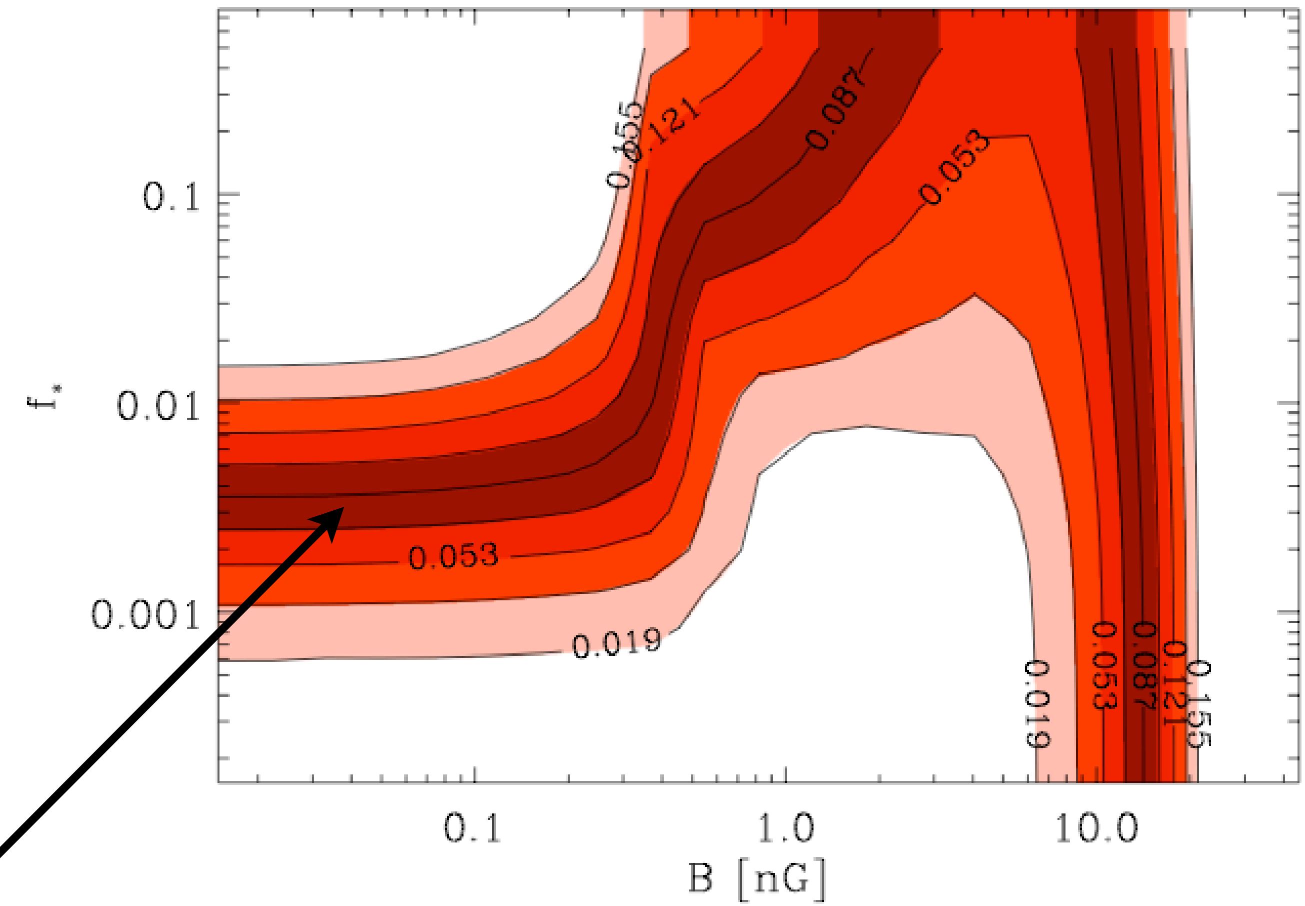
Schleicher et al. (2009)

Effects of primordial fields

Modification of primordial star formation
⟹ constraints from the optical depth



Reionization by Pop. III stars



Reionization by Pop. III & Pop. II stars

Schleicher, RB & Klessen (2008)

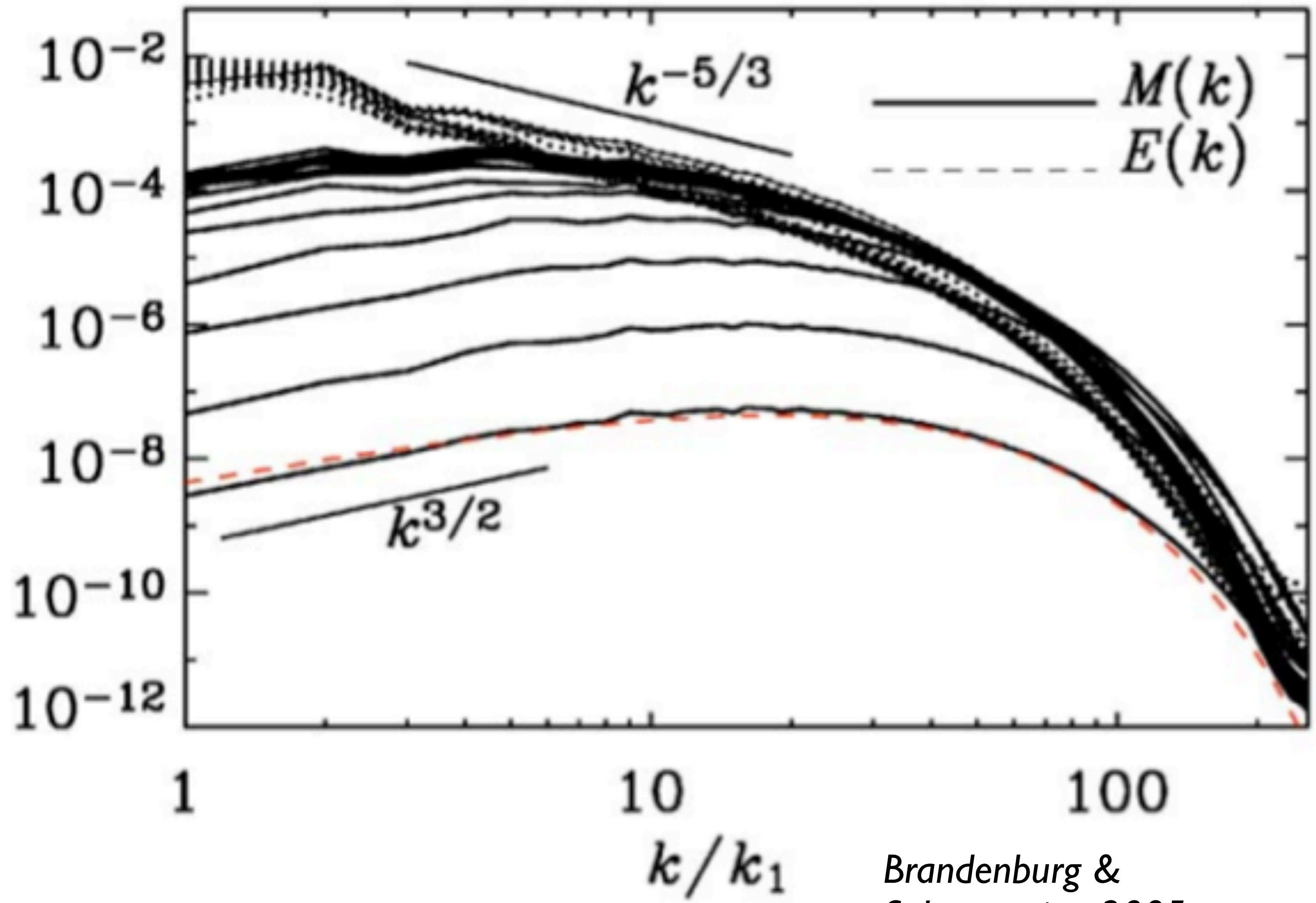
Reionization optical depth: 0.087 +/- 0.017

B-Field amplification

Small-scale dynamo

(Batchelor 1950, Kazantsev 1968, see also Brandenburg & Subramanian 2005 and Jennifer Schober's talk)

- exponential growth of weak seed fields
- growth rate depends on magnetic Reynolds number R_m : $\gamma \propto R_m^{-1/2}$
- mag. spectrum: $E_{mag,k} \propto k^{3/2}$
- saturation at $E_{mag} \sim 0.1\ E_{kin}$



Brandenburg & Subramanian 2005

B-Field amplification

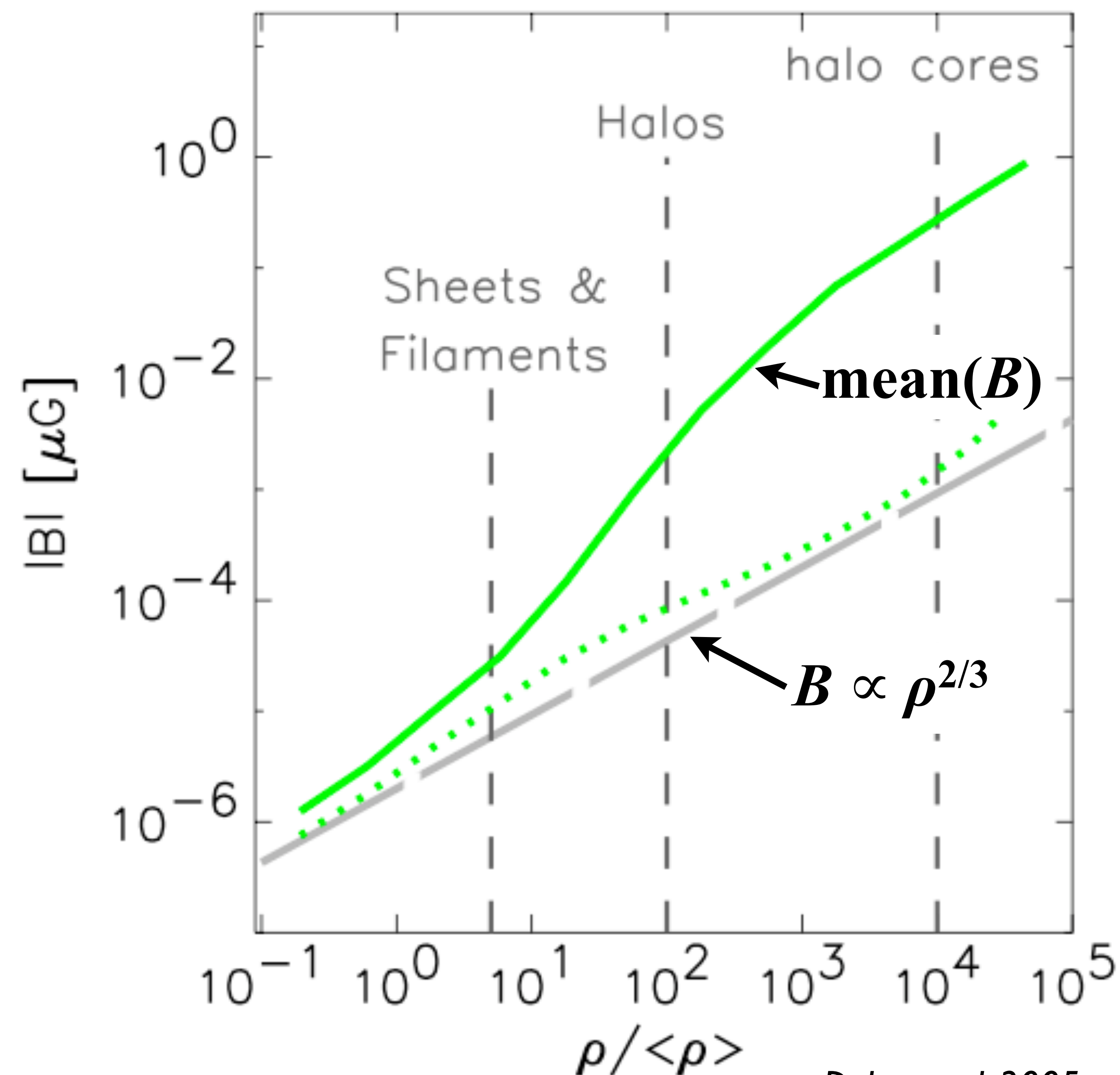
B-fields during compression

• maximum growth by adiabatic compression:

$B \propto \rho^{2/3}$

• small-scale dynamo works in cluster forming models (e.g. *Dolag et al. 1999, 2000; Xu et al. 2009, 2010*)

• depends on numerical resolution

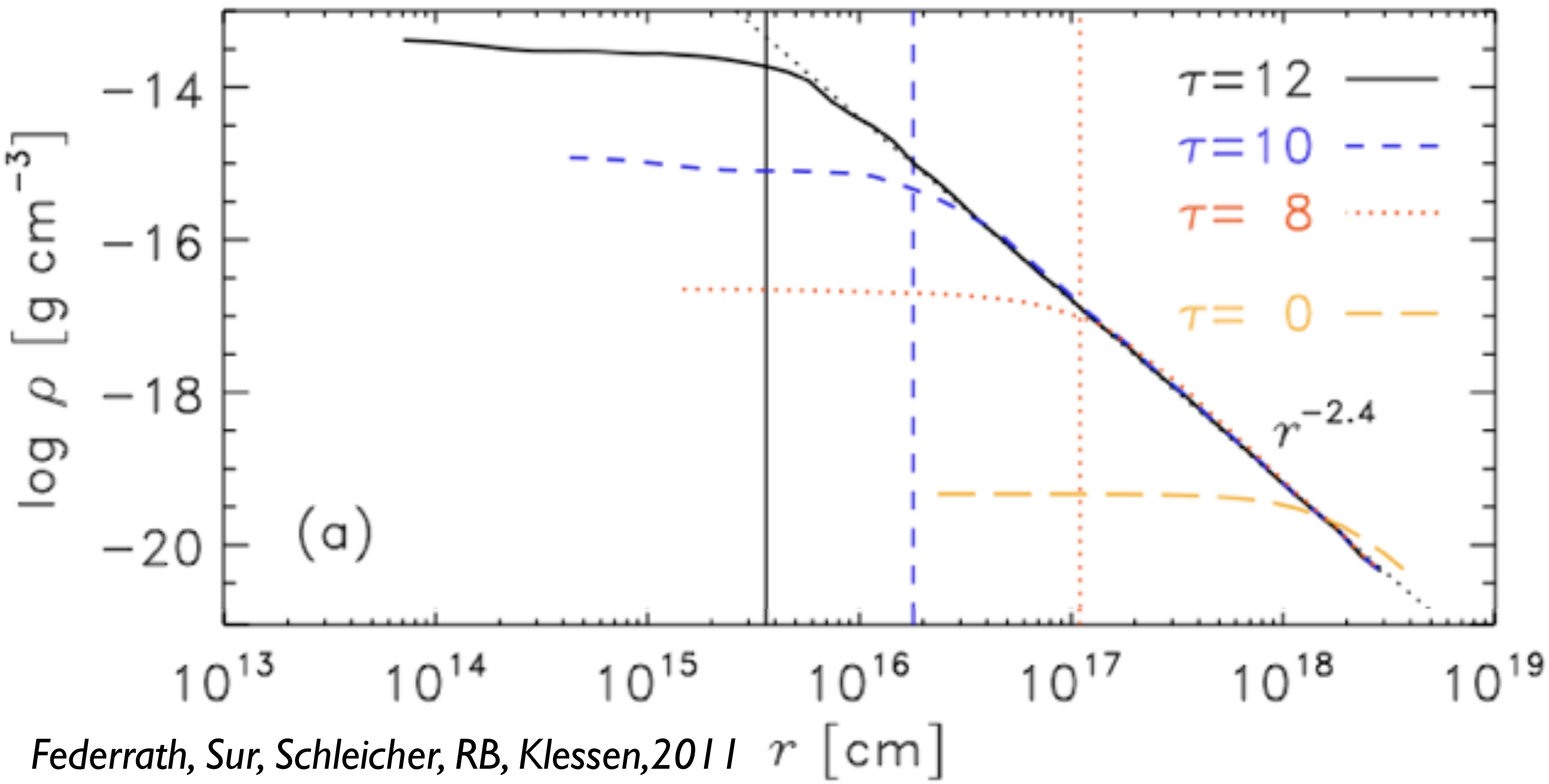


Dolag et al. 2005

Dynamo during "First Star Formation"

- turbulent infall motions (e.g. *Abel et al. 2002, Greif et al. 2008*)

- baryonic core modelled on a supercritical hydrostatic sphere:
  - $M_{\mathrm{baryon}} = 1500\ \mathrm{M_{sol}}$
  - $\rho_0 = 5\mathrm{x}10^{-20}\ \mathrm{g\ cm^{-3}}$
  - weak random field: $\mathrm{B} = 1\mathrm{nG},\ \beta = 10^{10}$
  - transonic turbulence: $\mathrm{v_{rms}} = 1.1\ \mathrm{km\ sec^{-1}}$



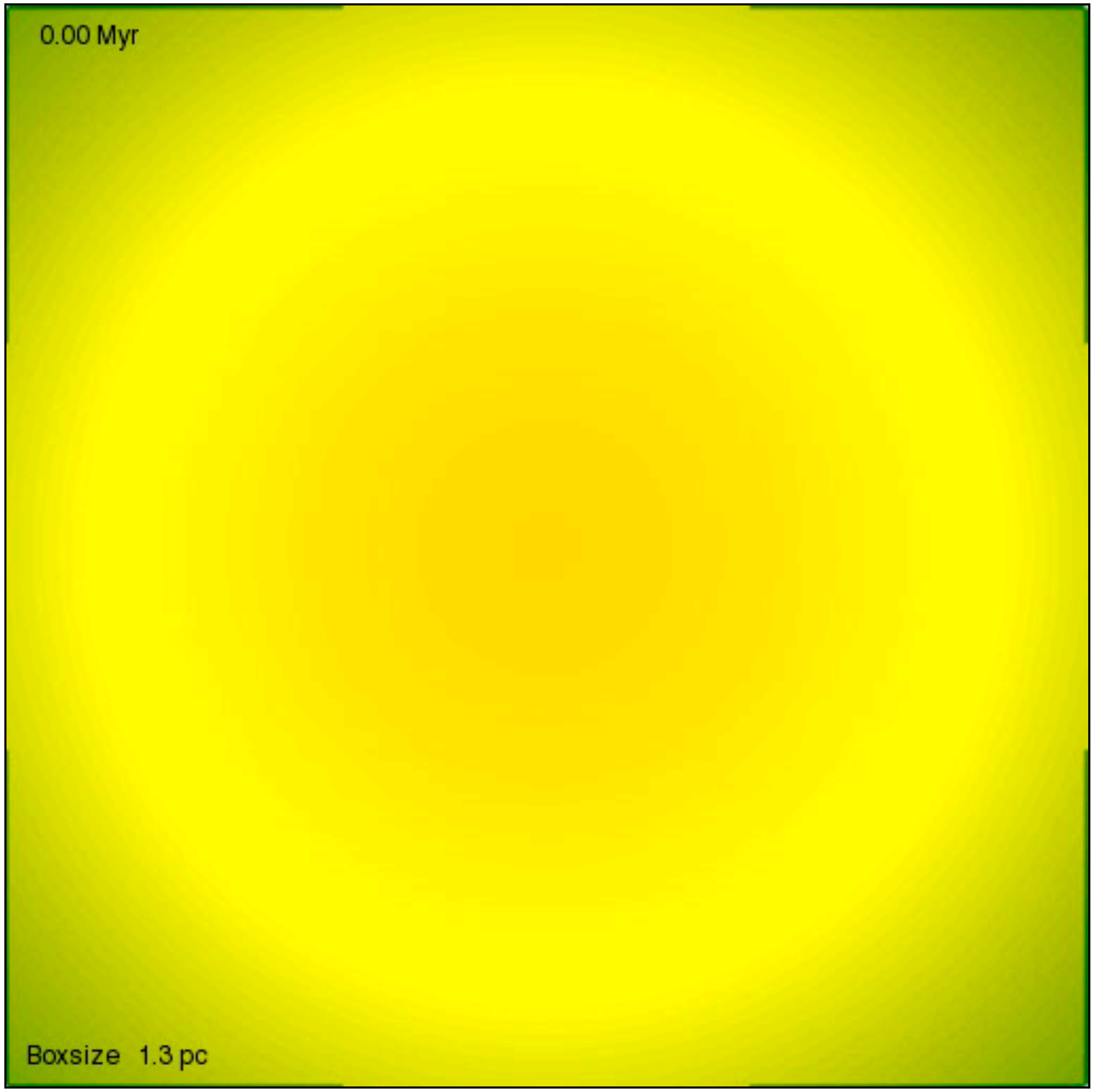
Federrath, Sur, Schleicher, RB, Klessen, 2011

characteristic length: **Jeans length**: $\lambda_J = \left(\frac{\pi\, c_s^2}{G\,\rho}\right)^{1/2}$

Dynamo during "First Star Formation"

- turbulent infall motions (e.g. *Abel et al. 2002*, *Greif et al. 2008*)

- baryonic core modelled on a supercritical hydrostatic sphere:
  - $M_{\mathrm{baryon}} = 1500\ \mathrm{M_{sol}}$
  - $\rho_0 = 5\mathrm{x}10^{-20}\ \mathrm{g\ cm^{-3}}$
  - weak random field: $\mathrm{B} = 1\mathrm{nG},\ \beta = 10^{10}$
  - transonic turbulence: $v_{\mathrm{rms}} = 1.1\ \mathrm{km\ sec^{-1}}$



characteristic length: **Jeans length**: $\lambda_{\mathrm{J}} = \left(\dfrac{\pi\, c_{\mathrm{s}}^2}{G\, \rho} \right)^{1/2}$

Dynamo during "First Star Formation"

- analyse data within decreasing Jeans volume:

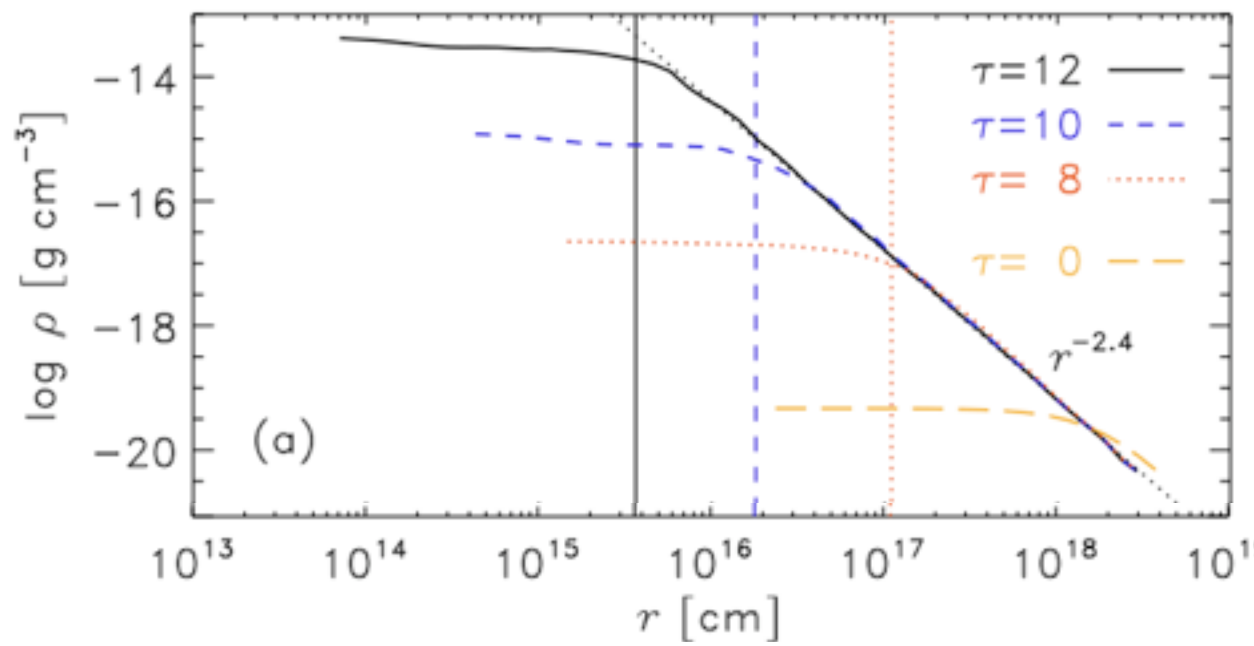
$$V_J = 4\pi(\lambda_J/2)^3/3$$

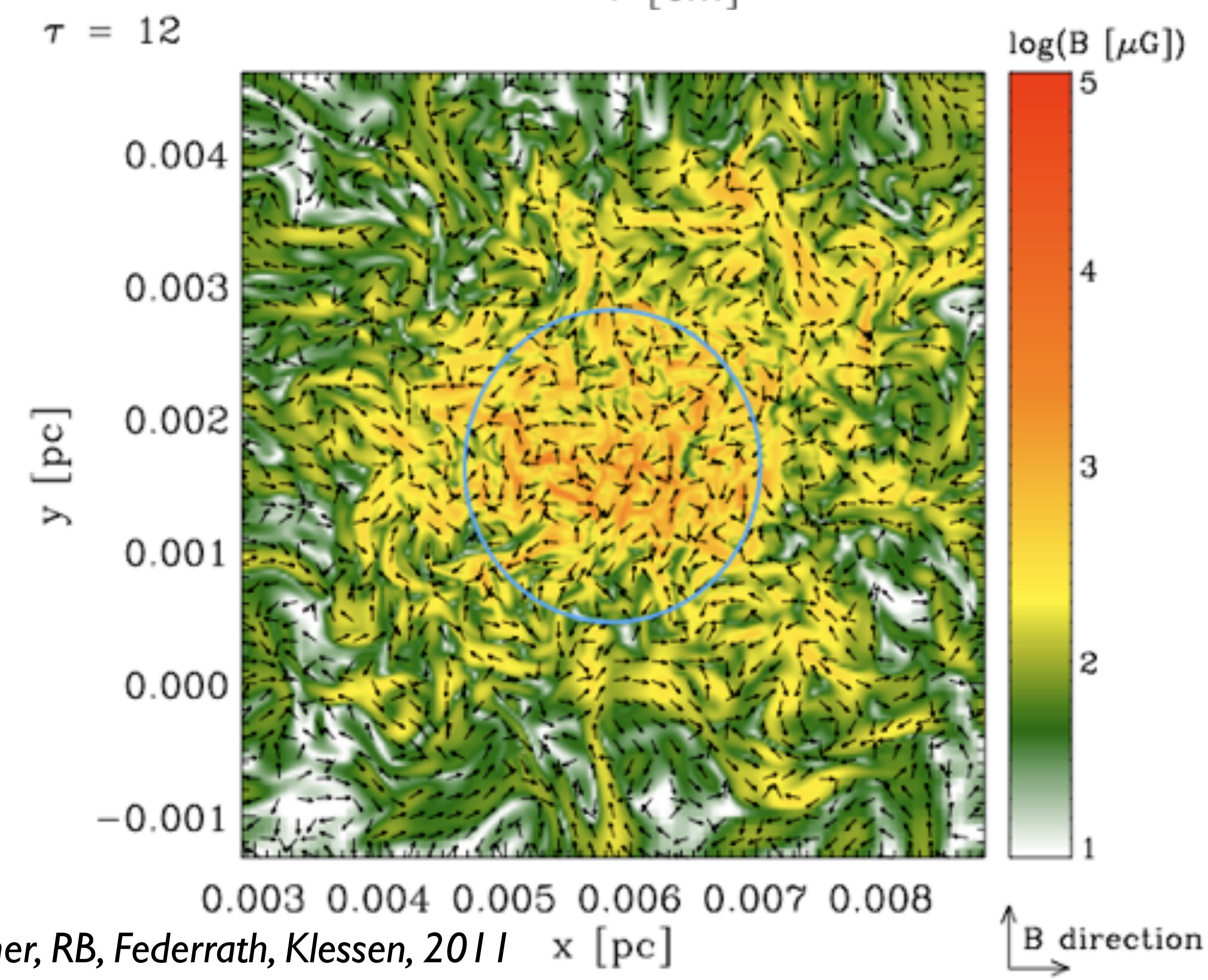
- use dimensionless time:

$$\tau = \int \frac{1}{t_{ff}(t)}\, dt$$

- free fall time

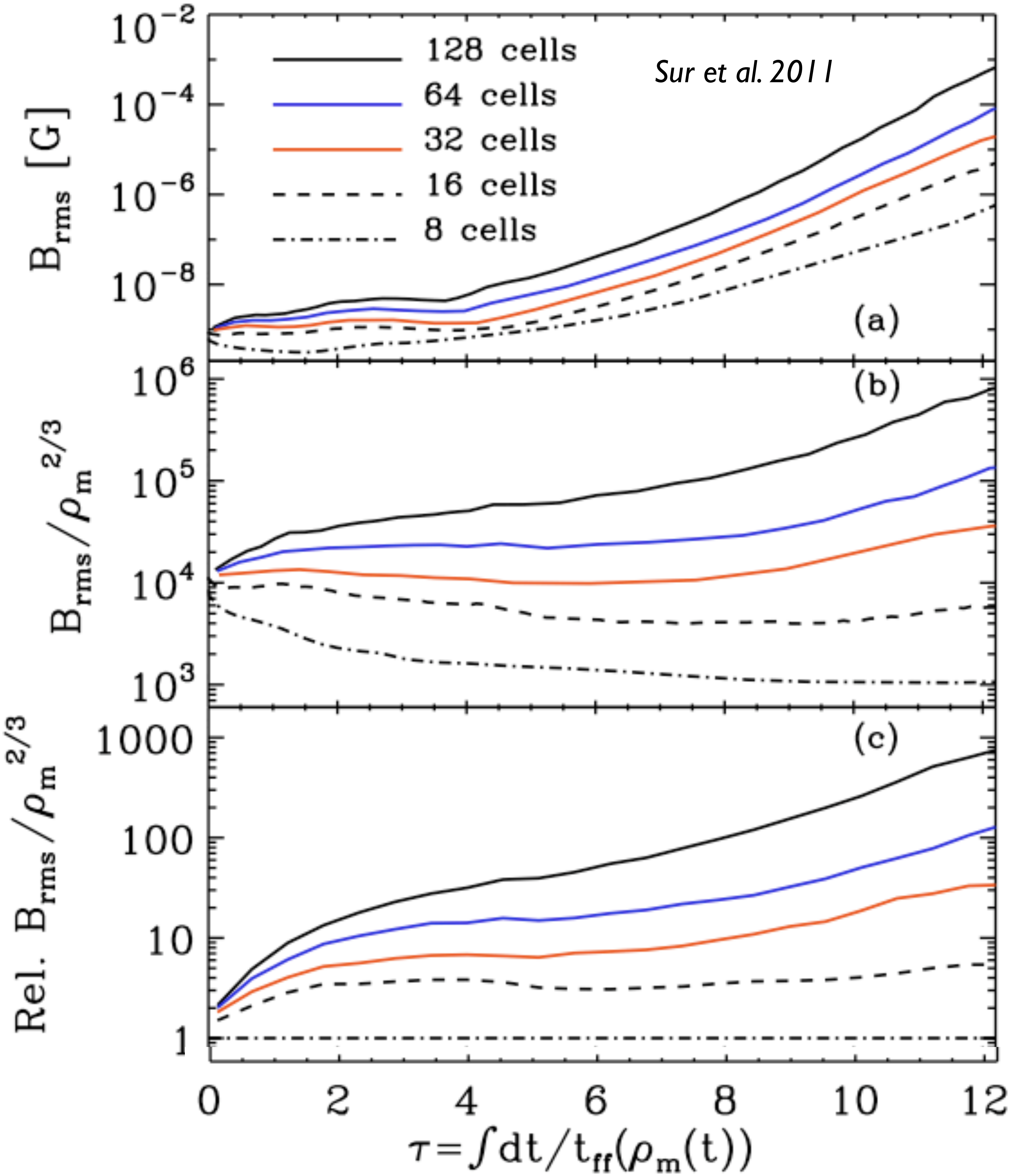
$$t_{ff} = (3\pi/32\, G\, \langle\rho(t)\rangle)^{1/2}$$





Sur, Schleicher, RB, Federrath, Klessen, 2011

Dynamo during "First Star Formation"



- growth rate depends on R_m, i.e. resolution:

$$R_m \propto N_J^{4/3}$$ (e.g. *Haugen et al. 2004*)

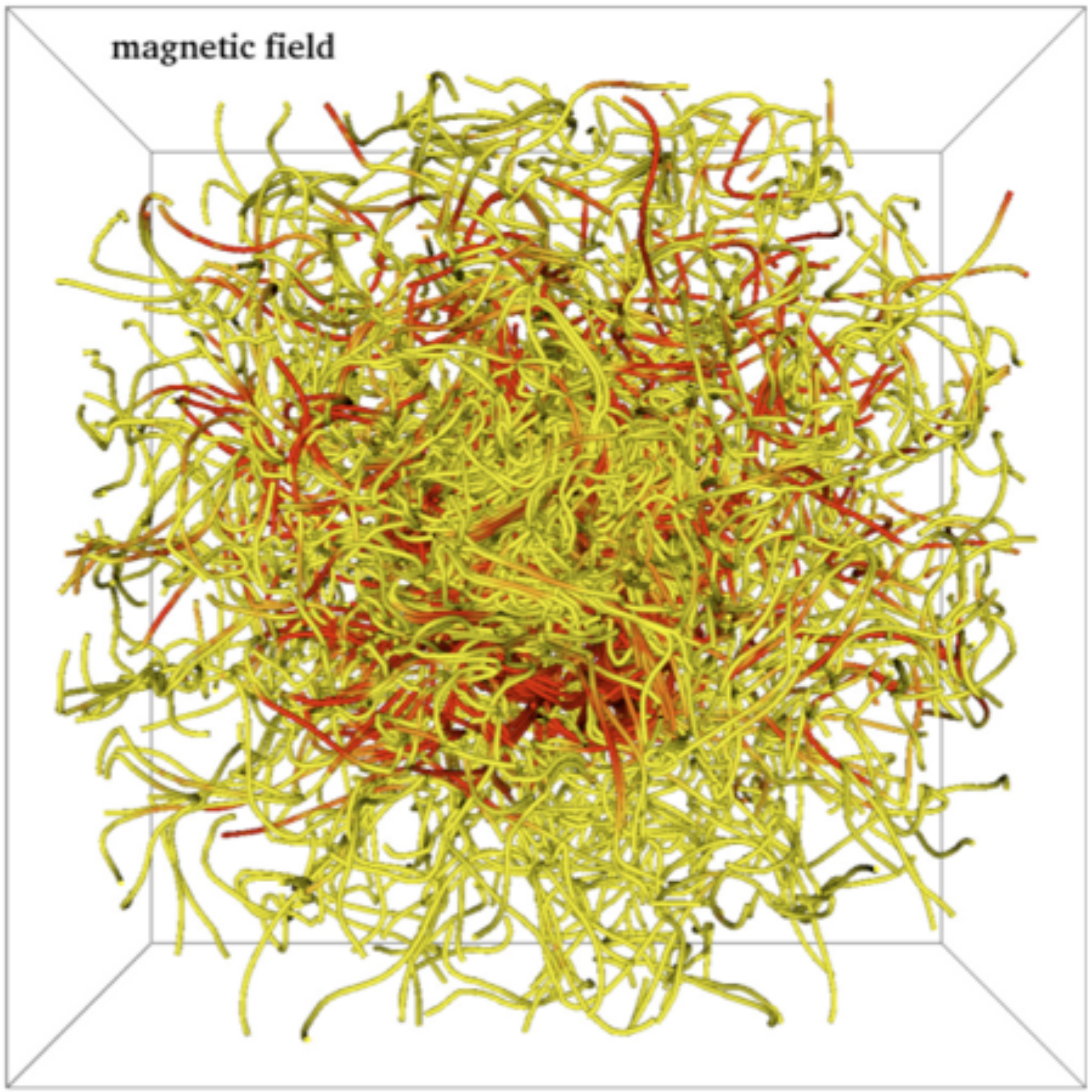
N_J: number of grid cells per local Jeans length; realization with adaptive mesh refinement (AMR)

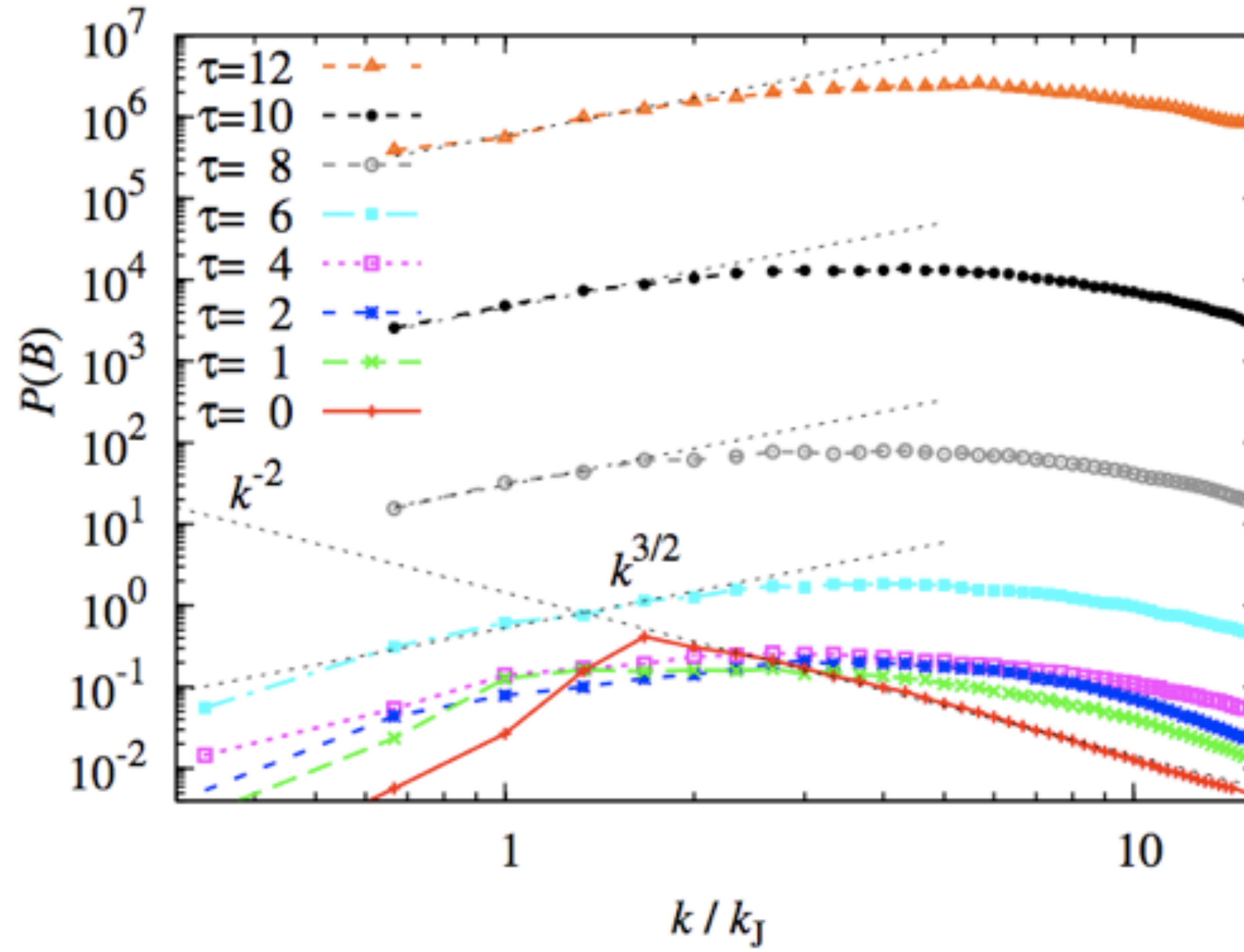
- minimum resolution: ~ 30 grid cells per Jeans length

Dynamo during "First Star Formation"

Magnetic field structure

Federrath et al.,2011

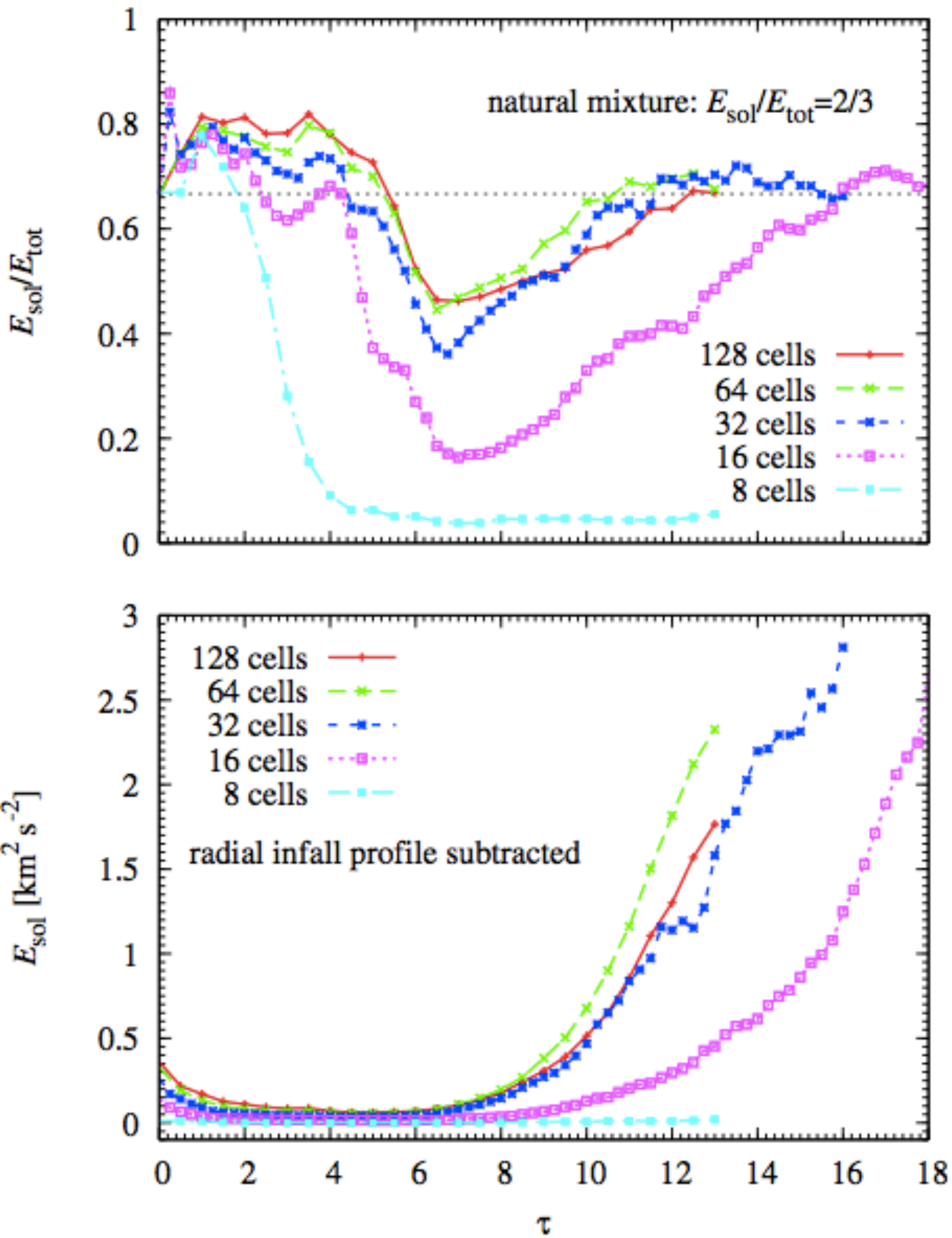


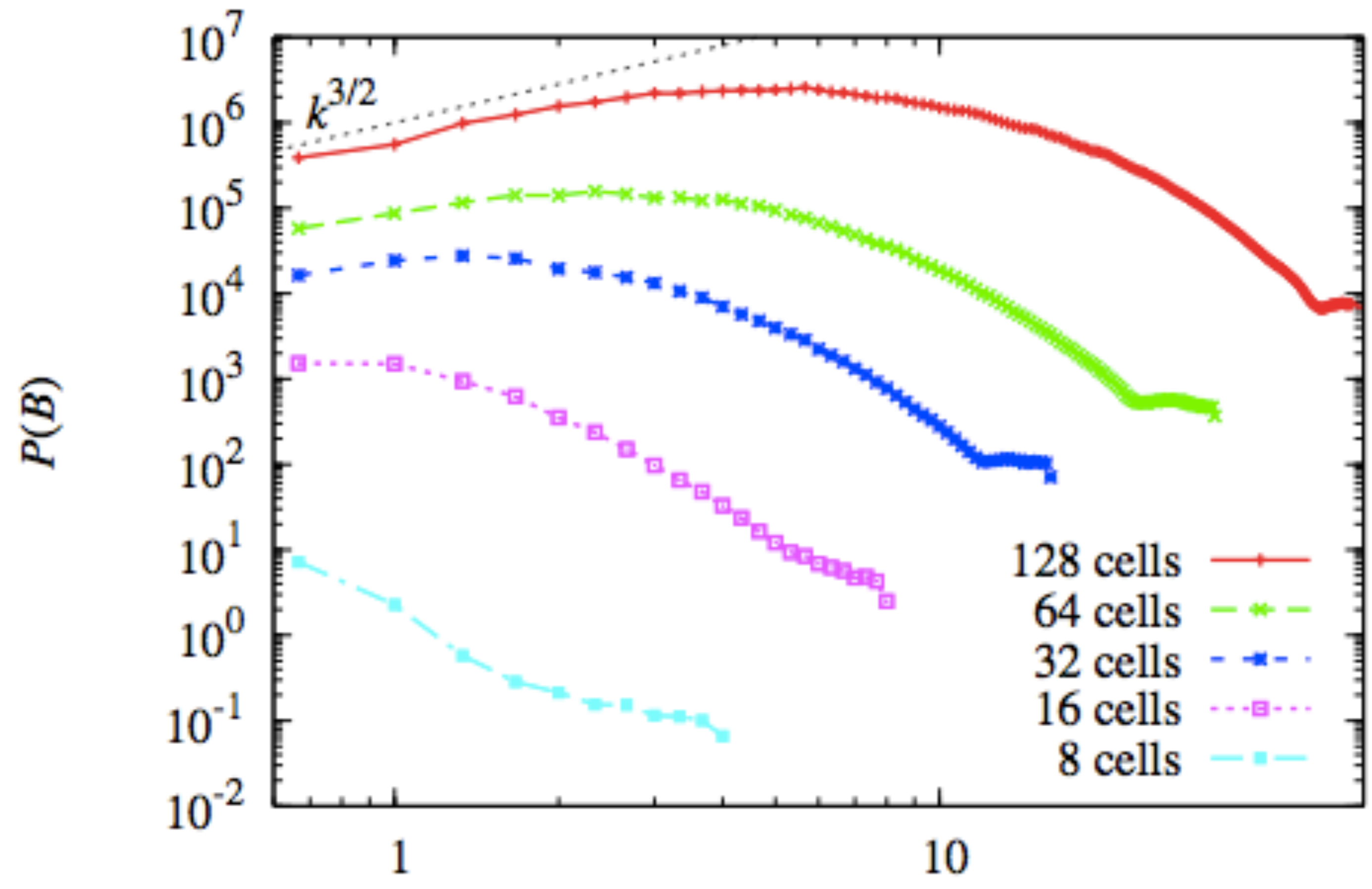


- spectra from numerical simulation follow Kazantsev theory

Dynamo during "First Star Formation"

Turbulence Properties & Magnetic Field spectra





Federrath et al.,2011

At least 30 cells per Jeans length to model turbulence and dynamos

Summary

- primordial magnetic fields can be strongly damped/ amplified during the cosmic evolution
- turbulent damping in the regime where $v_A \sim v$
- exponential amplification by the small scale dynamo when $v_A \ll v$ (works also very early epochs?)
- primordial fields influence thermal evolution of the early Universe (e.g. primordial star formation)

Don’t ignore Magnetic Fields!

The End